

Factoring and Solving Quadratic Equations

1. Factor completely.

a) $-3xyz - 21x^2yz$
 $= -3xyz(1 + 7x)$
 $= -3xyz(7x + 1)$

b) $25m^2 - 49 \Rightarrow (5m)^2 - (7)^2$
 $= (5m - 7)(5m + 7)$

c) $a^2 + 5a + 6$ M: $1 \times 6 = 6$
 $(a + 2)(a + 3)$ A: 5
 N: 2, 3

d) $6x^2 - 7x - 5$ M: $6 \times -5 = -30$
 $(2x + 1)(3x - 5)$ A: -7
 N: +3, -10

e) $64x^2 - 9y^2 = (7x)^2 - (3y)^2$
 $= (7x - 3y)(7x + 3y)$

f) $14m^2n + 7mn - 21n^2$
 $= 7n(2m^2 + m - 3n)$

g) $3m^2 - 5mn - 2n^2$ M: $3 \times -2 = -6$
 $(3m + 1)(m - 2)$ A: -5
 N: +1, -6

h) $x^2 - 6xy - 16y^2$ M: $1 \times -16 = -16$
 $(x + 2)(x - 8)$ A: -6
 N: +2, -8

2. Factor completely.

a) $4s^3 + 12s^2 - 16s$
 $= 4s(s^2 + 3s - 4)$ M: $1 \times -4 = -4$
 $= 4s(s - 1)(s + 4)$ A: 3
 N: -1, +4

b) $36x^3 - 84x^2 + 24x$
 $= 12x(3x^2 - 7x + 2)$ M: $3 \times 2 = 6$
 $= 12x(3x - 1)(x - 2)$ A: -7
 N: -1, -6

c) $4x^2 - 26x + 14$
 $= 2(2x^2 - 13x + 7)$ M: $2 \times -7 = -14$
 $= 2(2x + 1)(x - 7)$ A: -13
 N: +1, -14

d) $3a^2 + 18a + 27$
 $= 3(a^2 + 6a + 9)$ M: $1 \times 9 = 9$
 $= 3(a + 3)(a + 3)$ A: 6
 N: +3, +3

e) $5t^2 - 5t - 30$
 $= 5(t^2 - t - 6)$ M: $1 \times -6 = -6$
 $= 5(t + 1)(t - 6)$ A: -5
 N: +1, -6

f) $18x^2 + 15x - 12$
 $= 3(6x^2 + 5x - 4)$ M: $6 \times -4 = -24$
 $= 3(2x - 1)(3x + 4)$ A: 5
 N: -3, +8

Solutions

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|--------------------------|-------------------------|-----------------------|-----------------------|
| 1. a) $-3xyz(1 + 7x)$ | b) $(5m + 7)(5m - 7)$ | c) $(a + 3)(a + 2)$ | d) $(3x - 5)(2x + 1)$ |
| e) $(8x + 3y)(8x - 3y)$ | f) $7n(2m^2 + m - 3n)$ | g) $(m - 2n)(3m + n)$ | h) $(x - 8y)(x + 2y)$ |
| 2. a) $4s(s + 4)(s - 1)$ | b) $12x(x - 2)(3x - 1)$ | c) $2(x - 7)(2x + 1)$ | d) $3(a + 3)^2$ |
| e) $5(t - 3)(t + 2)$ | f) $3(2x - 1)(3x + 4)$ | | |

3. Solve by factoring.

a) $x^2 - 4x - 12 = 0$
 $(x+2)(x-6) = 0$
 $x = -2 \quad x = 6$
 The solution is $x = -2$
 $x = 6$
 M: $1 \times -12 = -12$
 A: -4
 N: $+2, -6$

b) $x^2 = 8x + 15 = 0$
 $(x-3)(x-5) = 0$
 $x-3=0 \rightarrow x=3$
 $x-5=0 \rightarrow x=5$
 M: $1 \times 15 = 15$
 A: -8
 N: $-3, -5$

c) $5x = 6x^2 + 1$
 $0 = 6x^2 - 5x + 1$
 $0 = (6x-2)(6x-3)$
 $6x-2=0 \rightarrow x=1/3$
 $6x-3=0 \rightarrow x=1/2$
 M: $6 \times 1 = 6$
 A: -5
 N: $-2, -3$

d) $50x^2 - 162 = 0$
 $50x^2 = 162$
 $x^2 = \frac{162}{50}$
 $x^2 = \frac{81}{25}$
 $x = \pm 1.8 \rightarrow x = 1.8 \quad x = -1.8$

$0 = (3x-1)(2x-1)$
 $3x-1=0 \rightarrow x=1/3$
 $2x-1=0 \rightarrow x=1/2$

4. Solve using the quadratic formula - exact roots.

a) $x^2 - 3x - 5 = 0$ $a=1 \quad b=-3 \quad c=-5$
 $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-5)}}{2(1)}$
 $= \frac{3 \pm \sqrt{9+20}}{2}$

$x_1 = \frac{3 + \sqrt{29}}{2}$, $x_2 = \frac{3 - \sqrt{29}}{2}$

b) $4y^2 + 7y - 9 = 0$ $a=4 \quad b=7 \quad c=-9$

$x_{1,2} = \frac{-7 \pm \sqrt{(7)^2 - 4(4)(-9)}}{2(4)}$

$x_{1,2} = \frac{-7 \pm \sqrt{49+144}}{8}$

$x_{1,2} = \frac{-7 \pm \sqrt{193}}{8}$

$x_1 = \frac{-7 + \sqrt{193}}{8}$, $x_2 = \frac{-7 - \sqrt{193}}{8}$

5. Solve using the quadratic formula - approximate roots, rounded to 2 decimal places.

a) $2j^2 + 5j - 4 = 0$ $a=2 \quad b=5 \quad c=-4$

$j_{1,2} = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-4)}}{2(2)}$

$j_{1,2} = \frac{-5 \pm \sqrt{25+32}}{4}$

$j_{1,2} = \frac{-5 \pm 7.55}{4}$
 $j_1 = -3.14$
 $j_2 = 0.64$

b) $3g^2 + 2 = 11g$ $a=3 \quad b=-11 \quad c=2$

$3g^2 - 11g + 2 = 0$

$g_{1,2} = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(3)(2)}}{2(3)}$

$g_{1,2} = \frac{11 \pm \sqrt{121-24}}{6}$
 $\rightarrow \sqrt{97} = 9.85$
 $g_1 = \frac{11 + 9.85}{6} = 3.48$
 $g_2 = \frac{11 - 9.85}{6} = 0.19$

Solutions

3. a) $x = 6, x = -2$

b) $x = 5, x = 3$

c) $x = \frac{1}{2}, x = \frac{1}{3}$

d) $x = \frac{9}{5}, x = -\frac{9}{5}$

4. a) $x = \frac{3 \pm \sqrt{29}}{2}$

b) $y = \frac{-7 \pm \sqrt{193}}{8}$

5. a) $j = 0.64, j = -3.14$

b) $g = 3.48, g = 0.19$