

ADDING AND SUBTRACTING RATIONAL NUMBERS

To add or subtract rational numbers, write them in an equivalent form with common denominators:

Ex1. $\frac{3}{5} + \frac{2}{7}$ use lowest common denominator (35)

$$= \frac{3 \times 7}{5 \times 7} + \frac{2 \times 5}{7 \times 5}$$

$$= \frac{21}{35} + \frac{10}{35}$$

add the numerators

$$= \frac{21 + 10}{35}$$

$$= \frac{31}{35}$$

check for possible further reductions

To add or subtract rational expressions we use the same steps:

Ex2. $\frac{2x - 3}{4} + \frac{3x - 1}{5} - \frac{x - 5}{2}$ LCD = 20

$$= \frac{(5)(2x - 3)}{(5)(4)} + \frac{(4)(3x - 1)}{(4)(5)} - \frac{10(x - 5)}{(10)(2)}$$

$$= \frac{(10x - 15)}{20} + \frac{(12x - 4)}{20} - \frac{(10x - 50)}{20}$$

$$= \frac{10x - 15 + 12x - 4 - 10x + 50}{20}$$

$$= \frac{12x + 31}{20}$$

Ex3. $\frac{5}{3x^2} - \frac{1}{2x} + \frac{3}{5x^3}$ LCD = $30x^3$

$$= \frac{5(10x)}{(3x^2)(10x)} - \frac{1(15x^2)}{(2x)(15x^2)} + \frac{3(6)}{(5x^3)(6)}$$

$$= \frac{50x - 15x^2 + 18}{30x^3}$$

Restriction: $x \neq 0$

$$= \frac{-15x^2 + 50x + 18}{30x^3}, x \neq 0$$

$$\text{Ex4. } \frac{3x-12}{x^2-x-12} - \frac{2}{x^2+6x+9} - \frac{1}{x^2-4x-21}$$

LCD = ??? We need to factor...!

$$= \frac{3(x-4)}{(x-4)(x+3)} - \frac{2}{(x+3)(x+3)} - \frac{1}{(x-7)(x+3)}$$

If we note restrictions, we can reduce individual fractions first

$$x \neq -3, 4, 7$$

$$= \frac{3}{(x+3)} - \frac{2}{(x+3)(x+3)} - \frac{1}{(x-7)(x+3)} \quad \text{LCD} = (x+3)(x+3)(x-7)$$

$$= \frac{3(x+3)(x-7)}{(x+3)(x+3)(x-7)} - \frac{2(x-7)}{(x+3)(x+3)(x-7)} - \frac{1(x+3)}{(x-7)(x+3)(x+3)}$$

$$= \frac{3(x+3)(x-7) - 2(x-7) - 1(x+3)}{(x-7)(x+3)(x+3)}$$

$$= \frac{3(x^2 - 4x - 21) - 2(x-7) - 1(x+3)}{(x-7)(x+3)(x+3)}$$

$$= \frac{3x^2 - 12x - 63 - 2x + 14 - x - 3}{(x-7)(x+3)(x+3)}$$

$$= \frac{3x^2 - 15x - 52}{(x-7)(x+3)(x+3)}, x \neq -3, 4, 7$$

Practice

$$1) \frac{5}{(x+1)(x+2)} - \frac{7}{(x+2)(x-4)}$$

Restrictions $x \neq -1, -2, 4$

LCD $(x+1)(x+2)(x-4)$

$$= \frac{5(x-4)}{(x+1)(x+2)(x-4)} - \frac{7(x+1)}{(x+1)(x+2)(x-4)}$$

$$= \frac{5x-20-7x-7}{(x+1)(x+2)(x-4)}$$

$$= \frac{-2x-27}{(x+1)(x+2)(x-4)} \quad x \neq -2, -1, 4$$

$$4) 7 + \frac{3m}{m-4} - \frac{m}{m+2}$$

Restriction $m \neq -2, 4$

LCD $(m-4)(m+2)$

$$= \frac{7(m-4)(m+2)}{1(m-4)(m+2)} + \frac{3m(m+2)}{(m-4)(m+2)} - \frac{m(m-4)}{(m+2)(m-4)}$$

$$= \frac{7(m^2 + 2m - 4m - 8) + 3m^2 + 6m - m^2 + 4m}{(m-4)(m+2)}$$

$$= \frac{7m^2 - 14m - 56 + 2m^2 + 4m + 6}{(m-4)(m+2)}$$

$$= \frac{9m^2 - 10m - 50}{(m-4)(m+2)} \quad m \neq -2, 4$$

2) $\frac{6}{x+4} + \frac{5}{x}$

Restrictions
 $x \neq 0, -4$

LCD = $(x)(x+4)$

$= \frac{6(x)}{(x+4)(x)} + \frac{5(x+4)}{x(x+4)}$

$= \frac{6x + 5x + 20}{x(x+4)}$

$= \frac{11x + 20}{x(x+4)} \quad x \neq -4, 0$

3) $\frac{7}{2y-1} = \frac{3}{1-2y}$

$= \frac{7}{2y-1} + \frac{3}{2y-1}$

$= \frac{10}{2y-1} \quad y \neq \frac{1}{2}$

5) $\frac{2x}{x-y} - \frac{3y}{x+y} + 1$

Restrictions $x \neq y, x \neq -y$

LCD $(x-y)(x+y)$
 $= \frac{2x(x+y)}{(x-y)(x+y)} - \frac{3y(x-y)}{(x+y)(x-y)}$

$= \frac{2x^2 + 2xy - 3xy + 3y^2}{(x-y)(x+y)}$

$= \frac{2x^2 - xy + 3y^2}{(x-y)(x+y)}$

$= \frac{(x+y)(2x-3y)}{(x-y)(x+y)}$

$= \frac{2x-3y}{x-y} \quad x \neq y, x \neq -y$

M: $2 \times 3 = 6$
 A: -1
 N: $+2, -3$

$\frac{(2x+2y)(2x-3y)}{2}$

6) $1 + \frac{2}{x} = \frac{3}{x^2}$
 $2 - \frac{1}{x} = \frac{1}{x^2}$

Restrict
 $x \neq 0$

① $\frac{1(x^2)}{1(x^2)} + \frac{2(x)}{x(x)} - \frac{3}{x^2}$
 $\frac{2(x^2) - 1(x)}{1(x^2) - x(x)} - \frac{3}{x^2} = \frac{x^2 + 2x - 3}{2x^2 - x - 3}$

② $\frac{x^2 + 2x - 3}{x^2 - 1}$

③ $= \frac{x^2 + 2x - 3}{2x^2 - x - 3} = \frac{(x-1)(x+3)}{(2x+1)(x-3)}$

M: $2 \times 3 = 6$
 A: -1
 N: $+2, -3$

$= \frac{(x-1)(x+3)}{(x+1)(x-3)} \quad x \neq -1, 3, 0$