Systems of Linear-Quadratic Equations

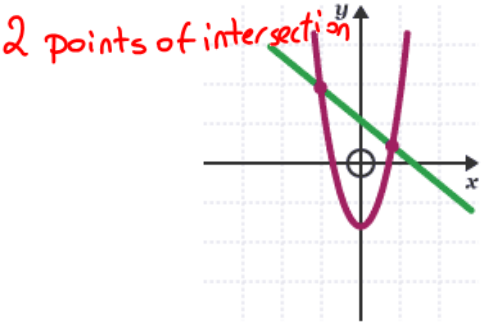
Recall:

The graph of a linear equation is a LINEAR.

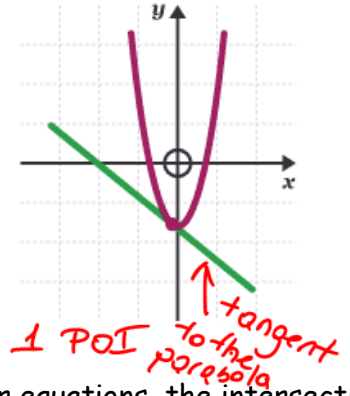
The graph of a quadratic equation is a PARABOLA

The diagrams below illustrate all the possible scenarios, in terms of intersection points, between a line and a parabola.

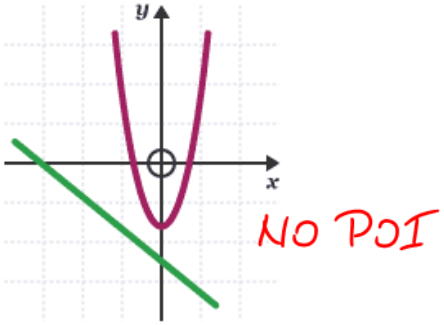
Scenario 1:



Scenario 2:



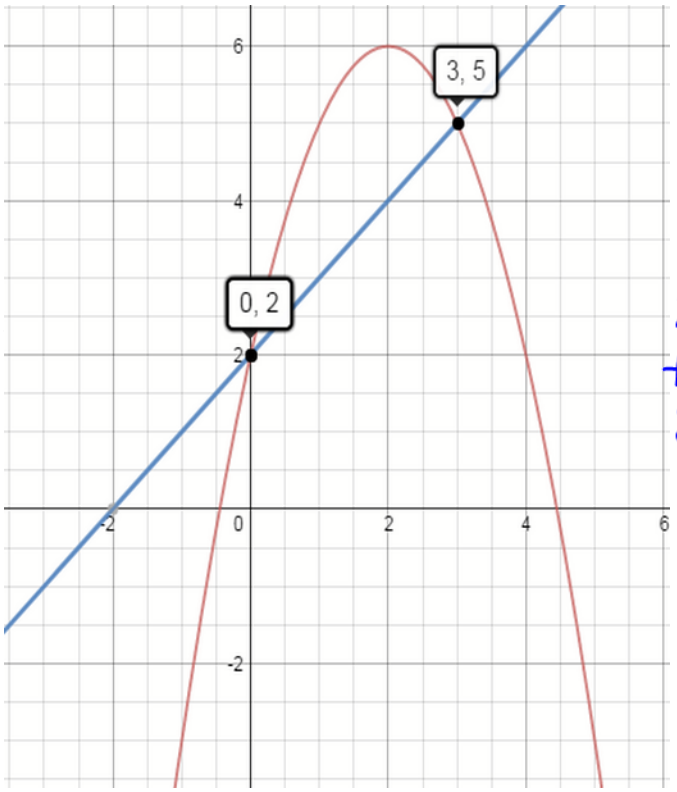
Scenario 3:



As in the case of a system of two linear equations, the intersection point(s) of a linear equation with a quadratic equation can be found graphically and/or algebraically.

Ex1. Find the point(s) of intersection of the given parabola and line. Solve graphically using desmos and algebraically.

a) $y = -x^2 + 4x + 2$ **and** $y = x + 2$



Sub ① into ②

$-x^2+4x+2 = x+2$

$0 = x^2-3x$

$0 = x(x-3)$

$x_1=0$ $x_2=3$

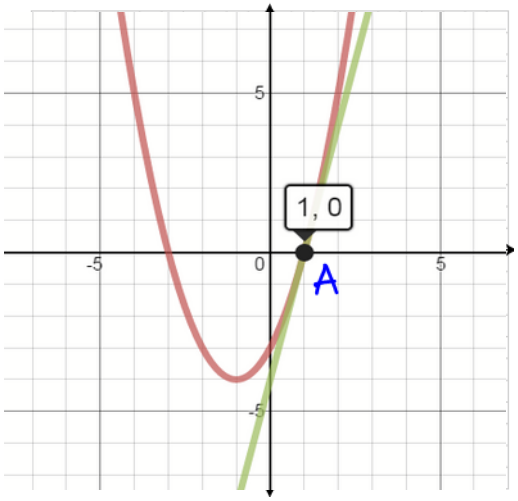
Now we need to find the "y" coordinate for each "x".

Sub "0" for x in any equation.

$x_1=0$ $y_1=x+2=2$ $A(0,2)$

$x_2=3$ $y_2=3+2=5$ $B(3,5)$

b) ① $y = x^2 + 2x - 3$ **and** ② $y = 4x - 4$



Sub ① into ②

$x^2+2x-3 = 4x-4$

$x^2-2x+1 = 0$

$(x-1)^2 = 0$

$\boxed{x=1}$

Sub "1" for "x" in any equation

$x=1 \quad y=(1)^2+2(1)-3 = 1+2-3 = 0$

$A(1,0)$

Ex2. Determine the number of points of intersection of $y = 3x^2 + 12x + 14$ and $y = 2x - 8$ without solving.

$3x^2+12x+14 = 2x-8$

$3x^2+10x+22 = 0$

$D = b^2-4ac = (10)^2 - 4(3)(22)$

$= 100 - 264$

$= -164$

$\therefore D<0$, therefore there're no POI

Ex3. The revenue equation for a company is $R(t) = -40t^2 + 300t$, where t is the ticket price in dollars. The cost equation is $C(t) = 1600 - 220t$. Determine the ticket price that will allow the company to break even.

① $P = R - C$

$0 = R - C$

$R = C$

② $-40t^2+300t = 1600-220t$

$0 = 40t^2-520t+1600$

$\frac{0}{40} = \frac{40(t^2-13t+40)}{40}$

$0 = t^2-13t+40$

$0 = (t-5)(t-8)$

$t=5 \qquad t=8$

∴ The ticket prices that would make the company break even are \$5 and \$8.

Ex4. Determine the value(s) of k such that the linear equation $y = -5x + k$ does not intersect the parabola $y = -2x^2 + 3x + 1$.

$-5x+k = -2x^2+3x+1$

$2x^2-8x+(k-1) = 0$

$b^2-4ac < 0$

$(-8)^2-4(2)(k-1) < 0$

$64-8k+8 < 0$

$72 < 8k$

$9 < k$

∴ k values must be greater than 9 for this system to have no POI.