

1. Solve each of the following using the **quadratic formula**. Round to two decimal places if necessary.

a. $x^2 - 2x + 1 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{0}}{2}$$

$$x = \frac{4}{2} = 2$$

$$\{2\}$$

b. $2g^2 - 3g + 11 = 0$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(11)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{-79}}{4}$$

no solutions

c. $2d^2 = -7d$

$$2d^2 + 7d = 0 \quad a=2 \quad b=7 \quad c=0$$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(2)(0)}}{2(2)}$$

$$= \frac{-7 \pm \sqrt{49}}{4} = \frac{-7 \pm 7}{4}$$

$$\{-3.5, 0\}$$

d. $(x-1)(x+3) = 6$

$$x^2 + 2x - 3 = 6$$

$$\therefore \{-4.15, 2.15\}$$

$$x^2 + 2x - 3 - 6 = 0$$

$$x^2 + 2x - 9 = 0$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-9)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{40}}{2} = \frac{-2 \pm 6.3}{2}$$

$$\left\{ \begin{array}{l} \frac{-2+6.3}{2} = 2.15 \\ \frac{-2-6.3}{2} = -4.15 \end{array} \right.$$

2. Solve each of the following by **factoring**.

a. $x^2 + 3x - 40 = 0$

M	A	N
-40	+3	-5, +8

$$(x-5)(x+8) = 0$$

$$\begin{array}{l} \downarrow \\ x-5=0 \\ \boxed{x=5} \end{array} \quad \begin{array}{l} \downarrow \\ x+8=0 \\ \boxed{x=-8} \end{array}$$

$$\{-8, 5\}$$

b. $4a^2 - 12a + 9 = 0$

$$4a^2 - 6a - 6a + 9 = 0$$

$$2a(2a-3) - 3(2a-3) = 0$$

$$(2a-3)(2a-3) = 0$$

M	A	N
36	+12	-6, -6

1	36
2	18
3	12
4	9

$$2a - 3 = 0$$

$$\frac{2a}{2} = \frac{3}{2}$$

$$\boxed{a = 3/2}$$

$$\left\{ \frac{3}{2} \right\}$$

c. $b^2 - 5b = 84$

$$b^2 - 5b - 84 = 0$$

$$(b+7)(b-12) = 0$$

$$\begin{array}{l} b+7=0 \\ \boxed{b=-7} \end{array} \quad \begin{array}{l} b-12=0 \\ \boxed{b=12} \end{array}$$

$$\{-7, 12\}$$

	84
1	84
2	42
3	28
4	21
7	12

d. $(x-2)(x+3) = 3(x+2) + 2x$

$$x^2 + x - 6 = 3x + 6 + 2x$$

$$x^2 + x - 6 - 5x - 6 = 0$$

$$x^2 - 4x - 12 = 0$$

$$(x+2)(x-6) = 0$$

$$\downarrow \\ x+2=0 \\ \boxed{x=-2}$$

$$\downarrow \\ x-6=0 \\ \boxed{x=6}$$

$$\{-2, 6\}$$

3. Determine the vertex of each parabola by completing the square:

a. $y = -3x^2 - 30x - 80$

$$\begin{aligned}
 &= -3(x^2 + 10x) - 80 \quad \frac{10}{2} = 5 \\
 &= -3(x^2 + 10x + 25 - 25) - 80 \quad (5)^2 = 25 \\
 &= -3(x^2 + 10x + 25) + 75 - 80 \\
 &= -3(x+5)^2 - 5 \\
 &\text{Vertex is } (-5, -5)
 \end{aligned}$$

b. $y = 4x^2 - 16x + 9$

$$\begin{aligned}
 y &= 4(x^2 - 4x) + 9 \quad -\frac{4}{2} = -2 \quad (-2)^2 = 4 \\
 &= 4(x^2 - 4x + 4 - 4) + 9 \\
 &= 4(x^2 - 4x + 4) - 16 + 9 \\
 &= 4(x-2)^2 - 7 \\
 &\text{Vertex is } (2, -7)
 \end{aligned}$$

c. $y = 4x^2 - 8x - 35$

$$\begin{aligned}
 &= 4(x^2 - 2x) - 35 \quad -\frac{2}{2} = -1 \quad (-1)^2 = 1 \\
 &= 4(x^2 - 2x + 1 - 1) - 35 \\
 &= 4(x^2 - 2x + 1) - 4 - 35 \\
 &= 4(x-1)^2 - 39 \\
 &\Rightarrow \text{Vertex is } (1, -39)
 \end{aligned}$$

d. $y = -2x^2 + 12x - 1$

$$\begin{aligned}
 &= -2(x^2 - 6x) - 1 \quad \frac{-6}{2} = -3 \quad (-3)^2 = 9 \\
 &= -2(x^2 - 6x + 9 - 9) - 1 \\
 &= -2(x^2 - 6x + 9) + 18 - 1 \\
 &= -2(x-3)^2 + 17 \\
 &\text{Vertex is } (3, 17)
 \end{aligned}$$

4. Determine the vertex of each parabola by averaging the zeros:

a. $y = x^2 + 7x - 30$

Step 1 $0 = (x-3)(x+10)$
 $x-3=0 \Rightarrow x=3$ $x+10=0 \Rightarrow x=-10$

Step 2 $V(x,y)$
 $x = \frac{-10+3}{2} = -6.5$

$y = (6.5-3)(6.5+10) = (3.5)(16.5) = 57.75$
 $\therefore \text{Vertex is } (-6.5, 57.75)$

c. $y = x^2 - 25$

Step 1 $0 = (x+5)(x-5)$
 $x+5=0 \Rightarrow x=-5$ $x-5=0 \Rightarrow x=5$

Step 2 $x = \frac{-5+5}{2} = 0$

$y = (0+5)(0-5) = -25$
 $\therefore V(0, -25)$

b. $y = 5x^2 - 15x$

Step 1 $0 = 5x(x-3)$
 $\frac{5x}{5} = 0 \Rightarrow x=0$ $x-3=0 \Rightarrow x=3$

Step 2 $V(x,y)$
 $x = \frac{0+3}{2} = 1.5$
 $y = 5x(x-3) = 5(1.5)(1.5-3) = -11.25$

$\therefore (1.5, -11.25)$

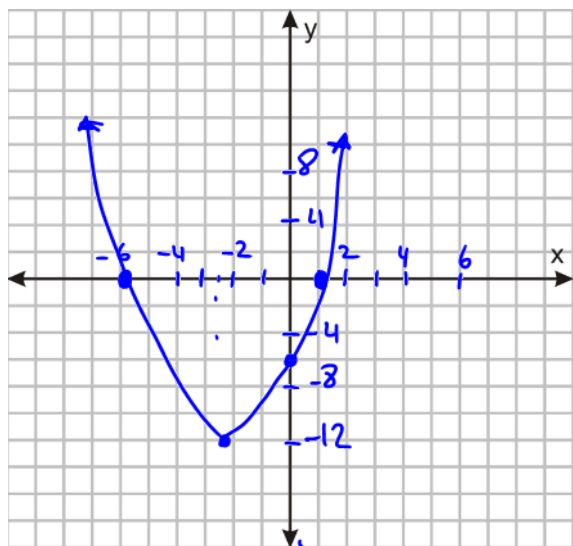
x	y	x
-2.0	+1	-4.5

d. $y = 2x^2 + x - 10$

$$\begin{aligned}
 0 &= 2x^2 - 4x + 5x - 10 \\
 0 &= 2x(x-2) + 5(x-2) \\
 0 &= (x-2)(2x+5) \\
 x-2=0 &\Rightarrow x=2 \\
 2x+5=0 &\Rightarrow 2x=-5 \Rightarrow x=-\frac{5}{2}
 \end{aligned}$$

$x = \frac{-2.5+2}{2} = -0.25$
 $y = (-0.25-2)(-0.25+2.5) = (-2.25)(2.25) = -10.125$
 $\therefore \text{Vertex is } (-0.25, -10.125)$

5. Find the **x intercepts**, **vertex**, and **y-intercept** of the quadratic relation below. Graph the relation on the grid provided without using a table of values. Label the axes and curve



Different scale for "y"

$$y = x^2 + 5x - 6$$

$$= (x^2 + 5x) - 6 \quad \frac{5}{2} = 2.5 \quad (2.5)^2 = 6.25$$

$$= (x^2 + 5x + 6.25 - 6.25) - 6$$

$$= (x + 2.5)^2 - 6.25 - 6$$

$$= (x + 2.5)^2 - 12.25$$

Vertex is $(-2.5, -12.25)$

Steps 1, 3, 5 b/c a is 1

$$0 = x^2 + 5x - 6 \quad \therefore \{-6, 1\}$$

$$0 = (x-1)(x+6) \quad \downarrow \quad \downarrow$$

$$x-1=0 \quad x+6=0 \quad x\text{-int}$$

$$\boxed{x=1} \quad \boxed{x=-6}$$

y-int $y = x^2 + 5x - 6$

6. Mrs. Mulock hits a golf ball off a tee. Its height above the ground can be approximated by using the equation $h = -4t^2 + 16t$, where h is the height above the ground in metres and t is the time in seconds. What is the maximum height of the golf ball? After how many seconds does this occur?

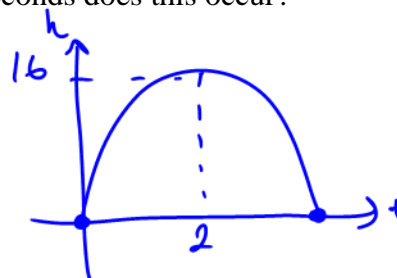
$$h = -4(t^2 - 4t) \quad \frac{-4}{2} = -2 \quad (-2)^2 = 4$$

$$= -4(t^2 - 4t + 4 - 4)$$

$$= -4(t^2 - 4t + 4) + 16$$

$$= -4(t-2)^2 + 16$$

Vertex is $(2, 16)$



\therefore The ball reach its max height of 16m in 2 sec.

7. Mr. Mulock works on weekends selling thingamajigs. The profit he earns is determined by the equation $P = -3n^2 + 72n - 310$, where n is the number of thingamajigs sold and P is the profit in dollars.
- What is the maximum profit that Mr. Mulock can earn?
 - Is it possible for Mr. Mulock to obtain a profit of \$200? Show all work to justify your answer.

a. $P = -3(n^2 - 24n) - 310$

$$= -3(n^2 - 24n + 144 - 144) - 310$$

$$= -3(n-12)^2 + 432 - 310$$

$$= -3(n-12)^2 + 122$$

\therefore The max profit is \$122

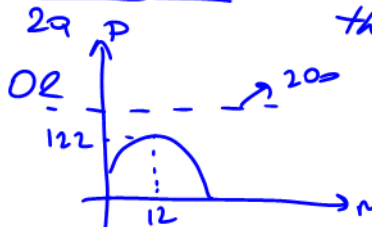
b. $200 = -3n^2 + 72n - 310$

$$3n^2 - 72n + 310 + 200 = 0$$

$$3n^2 - 72n + 510 = 0$$

$$x = \frac{-(-72) \pm \sqrt{(-72)^2 - 4(3)(510)}}{2(3)}$$

discriminant is negative therefore no solutions.



8. When Mr. Math realized that he had superpowers, he decided to try them out. He took a basketball and threw it into the air. Mrs. Math watched Mr. Math throw the ball and saw that it took the shape of a parabola with its height, 'h' in metres, being related to its time in the air, 't' seconds, by the equation $h = -5t^2 + 60t + 2$.
- If the ball's starting position was at the top of Mr. Math's head, how tall is Mr. Math?
 - What is the maximum height of the ball?
 - When does the ball reach its maximum height?
9. A farmer's field borders a stream. The farmer has 300 m of fencing to build a rectangular enclosure alongside the stream (so he only needs to build three sides). The area of the field is given by $A = w(300 - 2w)$ where w is the length of the two matching sides. Draw a labelled diagram to represent the enclosure, and find the maximum area the farmer could enclose.
10. The backboard behind a basketball net is a rectangle whose area can be represented by the trinomial $A = 3x^2 + 35x + 22$ (remember, the area of a rectangle is $A = (L) \times (W)$). Determine the two expressions that represent the length and width.

11. Astronauts have performed various experiments while on the moon. In one of their experiments, a projectile was launched and observed. The projectile reached a maximum height of 21 m and landed on the surface of the moon after 3.8 sec. Select the equation for the height, h , in metres, of the projectile after t seconds.

a. $h = -5(t - 3.8)^2 + 21$

b. $h = -5(t - 1.9)^2 + 21$

c. $h = -5(t + 3.8)^2 + 21$

d. $h = -5(t + 1.9)^2 + 21$

12. A water balloon was tossed in the air, and of course Mr. Culhane came up with a quadratic equation that models the total horizontal distance (in metres) covered throughout the flight of the balloon. The equation he came up with was $-0.6d^2 + 3.25d = -7.1$. Determine the horizontal distance covered by this toss by solving the equation above. Round your answer to two decimal places.