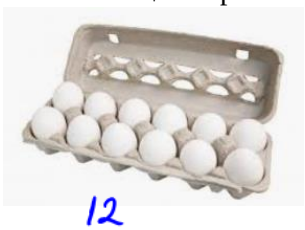
MPM1D **Date:**

Day 11: Rates, Ratios and Proportions **Unit 1: Number Sense**

RATE

Problem 1: Which carton of eggs (same brand and size) would you advise your family to buy? Which is the better buy economically?

STORE A: $2.64 per carton of eggs



12

Cost of 1 egg $= \frac{\$2.64}{12 \text{ eggs}} = \frac{\$0.22}{1 \text{ egg}}$

STORE B: $4.50 per carton of eggs

18

Cost of 1 egg $= \frac{\$4.50}{18 \text{ eggs}} = \frac{\$0.25}{1 \text{ egg}}$

Conclusion: 12 egg carton is cheaper by $0.03 or 3¢ per one egg.

DEFINITION: Rate is a comparison of two related numbers (quantities) having **different units**. A rate is usually written as a 'unit rate', in which the second term is always one like in the example above: 22 cents per one egg.

Example 1: Wage
John earns $60 for working 4 hours. What is his **rate of pay**?

Rate of pay $= \frac{\$60}{4 \text{ hours}} = \frac{\$15}{1 \text{ hr}}$

Example 2: Speed
If it takes 2 hours to travel 180 km distance, what is the rate of change (speed)?

Rate of Change $= \frac{180 \text{ km}}{2 \text{ h}} = \frac{90 \text{ km}}{1 \text{ h}}$

Example 3: Unit Cost
A 200g bag of mixed nuts costs $3.40. Calculate the **unit rate**.

Unit price $= \frac{\$3.40}{200 \text{ g}} = \frac{\$0.017}{1 \text{ g}}$

PROPORTION

Proportion is an equation, which states that two fractions are equal.

Example 1

$\frac{1}{2} = \frac{m}{4}$ (×2 on top, ×2 on bottom)

$m = 1 \times 2$

$m = 2$

Example 2

$\frac{5}{m} = \frac{25}{15}$ (÷5 on top, ÷5 on bottom)

$m = 15 \div 5$

$m = 3$

Example 3 * *cross multiplication*

Step1 $\frac{3}{5} = \frac{8}{m}$ Step2 $\Rightarrow 3m = 5 \cdot 8$ (multiply cross with circle squares with square) Step3 $\Rightarrow \frac{3m}{3} = \frac{40}{3}$ (divide both sides by 3) $\Rightarrow m \cong 13.3$

Problem 2: John's family consumes 4 eggs a day. At this rate, how many eggs would they consume in one year? (Set up a proportion that shows the daily rate = yearly rate)

daily rate = yearly rate

Step1 $\frac{4 \text{ eggs}}{1 \text{ day}} = \frac{? \text{ eggs}}{365 \text{ days}}$ Step2 $\Rightarrow$ Cross multiply: $4 \cdot 365 = 1 \cdot n$

Replace ? with letter "n"

$1460 = n$

∴ They consume 1460 eggs in one year.

Problem 3: How much would John's family save in a year if they chose the 12-carton egg? (Set up a proportion that shows amount saved per one egg = amount saved per total eggs in one year)

Step1 $\frac{\$0.03}{1 \text{ egg}} = \frac{?}{1460 \text{ eggs}}$ $\Rightarrow$ Cross multiply $\frac{0.03}{1} = \frac{s}{1460}$ $\Rightarrow 0.03 \cdot 1460 = 1 \cdot s$

$43.8 = s$

Replace ? with letter "s" for savings

∴ They would save $43.80 in one year.

Example 1: Recipe
The recipe calls for 1 cup of rye flour to 2 cups of all-purpose flour. How many cups of rye flour would you need to use if you ~~addd~~ 6 cups of all-purpose flour?

$\frac{1 \text{ cup rye}}{2 \text{ cups reg}} = \frac{?}{6}$

$? = 3$

∴ 3 cups of rye flour.

Example 2: Weight
A rope's length and weight are in proportion. When 20m of rope weighs 1 kg, then 200 m of rope weighs:

$\frac{1\text{kg}}{20\text{m}} \overset{\times 10}{=} \frac{?}{200\text{m}}$

$? = 1 \times 10$
$= 10\text{kg}$.

∴ It weighs 10 kg.

Example 3: Speed
A pendulum completes 7 swings every three seconds. How many swings does it complete in a minute? 1 min = 60 sec

$\frac{7 \text{ swings}}{3 \text{ sec}} \overset{\times 20}{=} \frac{?}{60 \text{ sec}}$

$? = 7 \times 20$
$= 140$ swings

∴ It completes 140 swings.

Example 4: Price (cost)
Apples are $2.00 per dozen (12), how many apples can you get for $5.50?

$\frac{\$2.00}{12 \text{ apples}} = \frac{\$5.50}{a}$

$2 \times a = 12 \times 5.50$
$\frac{2 \times a}{\div 2} = \frac{66}{\div 2}$
$a = 33$

∴ You'd get 33 apples.

RATIO

Ratio is a comparison of two quantities **with the same units.**

1 yellow square to 2 blue squares

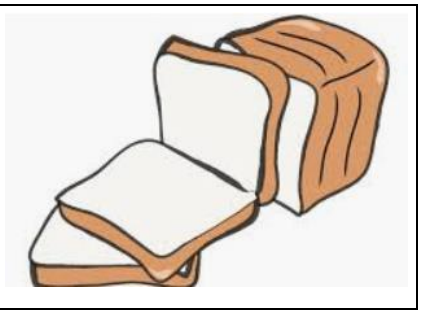
We express ratios in three different ways:

- use ":" to separate the values → 1 : 2
- use the word "to" → 1 to 2
- write like a fraction → ½

Bilal uses the following ingredients for his favorite bread:

- 1 cup of rye flour
- 2 cups of all-purpose flour
- 1 cup of milk

What is the ratio of rye flour to all-purpose flour? 1 : 2 1 to 2 1/2
What is the ratio of rye flour to total flour? 1 : 3 1 to 3 1/3
What is the ratio of all-purpose flour to total flour? 2 : 3 2 to 3 2/3
What is the ratio of total flour to milk? 3 : 1 3 to 1 3/1



Write each ratio in simplest form. Find GCF, and then divide each quantity by the GCF.

a. $\frac{6}{15}$ GCF = 3 → $\frac{2}{5}$

b. 4 : 12 : 16 GCF = 4
÷4 ÷4 ÷4
1 : 3 : 4

c. 6 to 10 GCF = 2
÷2 ÷2
3 to 5

Practice: Ratios, Rates, and Proportions

Write the following as ratios in lowest terms

a. 73 days to 1 year

73 to 365

1 to 5

b. 35cents to $1.05

35 to 105

1 to 3

c. 750 mL to 1.5 L

750 to 1500

1 to 2

d. 3 min to 45 sec

×60

180 to 45

4 to 1

Find the unit rate of the following:

e. Mike earns $42 in 6 hours.

$= \frac{\$47}{6h}$

$= \frac{\$7}{1h}$

f. $350 for 8 people to attend a party

$= \frac{\$350}{8p}$

$= \frac{\$43.75}{1p}$

g. 24 pop for $6.96

$= \frac{\$6.96}{24}$

$= \frac{\$0.29}{1 pop}$

Answer the following ~~rate~~ problems:

h. Jack earned $50 in 10 hours, while John earned $105 in 20 hours. Which person had the better rate of pay?

Jack: $\frac{\$50}{10h} = \frac{\$5}{1h}$

John: $\frac{\$105}{20h} = \frac{\$3.25}{1h}$

∴ John has made extra 25¢ per

i. A bus travels 10 km in 25 minutes. At this rate, how far will the bus travel in one hour?

$\frac{10\text{ km}}{25\text{ min}} \times \frac{?\to d}{60\text{ min}}$

$10 \cdot 60 = 25 \cdot d$

$600 = 25 \cdot d$

$\div 25 \quad \div 25$

$24 = d$

∴ 24 km in one hour.

j. Oranges are $2.00 per dozen. At this rate, how many oranges could you get for $3.50

$\frac{\$2.00}{12} \times \frac{\$3.50}{?\to a}$

$2 \times a = 12 \times 3.50$

$2 \times a = 42$

$\div 2 \quad \div 2$

$a = 21$

∴ 21 apples.

k. Katherine cycled 30 km in 2 hours. If she continues at the same rate, what distance will she travel in 7 hours?

$\frac{30\text{km}}{2\text{hr}} : \frac{d}{7\text{hr}}$

$30 \times 7 = 2 \times d$

$210 = 2 \times d$

$\div 2 \quad \div 2$

$105 = d$

∴ She'll travel 105 km in 7 hours.

l. Which is the better value? $350 for a bus of 35 people, or $440 for a bus of 40 people?

$\frac{\$350}{35p} = \frac{\$10}{1p}$ ← better value

$\frac{\$440}{40p} = \frac{\$11}{1p}$

m. Which is the better value? 28 g of mixed nuts for $0.84, or 35g of mixed nuts for $1.40?

$\frac{\$0.84}{28g} = \frac{\$0.03}{1g}$

$\frac{\$1.40}{35g} = \frac{\$0.04}{1g}$

better. Paying $0.01 less.

Find the missing value in the following proportions *round to 2d.p. where necessary

n. $\frac{3}{8} = \frac{m}{5}$

$15 = 8m$

$\div 8 \quad \div 8$

$\frac{15}{8} = m$

$m \cong 1.88$

o. $\frac{2}{k} = \frac{11}{4.5}$

$9 = 11 \cdot k$

$\div 11 \quad \div 11$

$0.82 = k$

p. $\frac{1.2}{2.8} = \frac{3}{p}$

$1.2p = 8.4$

$\div 1.2 \quad \div 1.2$

$p = 7$

q. $\frac{5}{3.2} = \frac{2.5}{y}$

$5 \cdot y = 8$

$\div 5 \quad \div 5$

$y = 1.6$

ANSWERS: a. 1:5, b. 1:3, c. 5:6, d. 4:1, e. $7/h, f. $43.75/p, g. $0.29/pop, h. John, i. 24km/h, j. 21oran., k. 105 km, l. $350/35, m. 28g/$0.84, n. m=1.88, o. k=0.82, p. p=7, q. y=1.6