SEQUENCES

A **sequence** is a list of numbers arranged in an order.

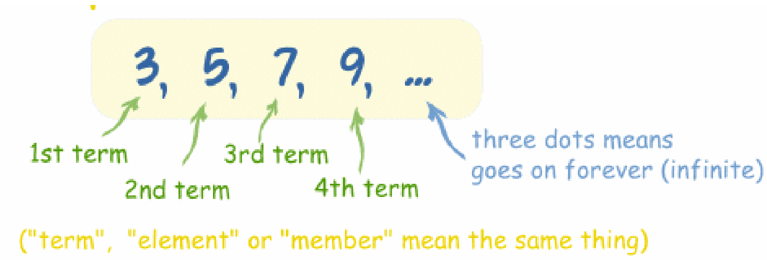
The numbers in a sequence are called **terms**

The first term is referred to as t_1, the second as t_2, and so on.

Infinite or Finite

A sequence that has a last term is a **finite sequence.** A sequence that has no last term and continues indefinitely is an **infinite sequence.**

The ***n*^th^ term** is referred to as t_n (or the **general term** of the sequence), where n is a natural number referring to the position of the term in the sequence.



Adapted from mathisfun.com

Examples:

{1, 2, 3, 4, ...} is a very simple sequence (and it is an **infinite sequence**)

{20, 25, 30, 35, ...} is also an infinite sequence

{1, 3, 5, 7} is the sequence of the first 4 odd numbers (and is a **finite sequence**)

{4, 3, 2, 1} is 4 to 1 **backwards**

{1, 2, 4, 8, 16, 32, ...} is an infinite sequence where every term doubles

{a, b, c, d, e} is the sequence of the first 5 letters **alphabetically**

{f, r, e, d} is the sequence of letters in the name "fred"

{0, 1, 0, 1, 0, 1, ...} is the sequence of **alternating** 0s and 1s (yes they are in order, it is an alternating order in this case)

Ex1. Given the formula for the n^{th} term, determine the first **three** terms of each sequence.

a) $t_n = 3n^2 - 4$

$t_1 = 3(1)^2 - 4 = 3 - 4$

$\boxed{t_1 = -1}$

$t_2 = 3(2)^2 - 4$

$\boxed{t_2 = 8}$

$t_3 = 3(3)^2 - 4$

$= 27 - 4$

$\boxed{t_3 = 23}$

b $f(n) = \frac{n-2}{n+2}$

$f(1) = \frac{1-2}{1+2} = -1/3$

$f(2) = \frac{2-2}{2+2} = 0/4 = 0$

$f(3) = \frac{3-2}{3+2} = 1/5$

ARITHMETIC SEQUENCES

In an **arithmetic sequence** the difference between consecutive terms is constant.

This constant is the **common difference, *d*.**

The **first term, t_1,** is denoted by the letter ***a*.**

Ex2. Find the next three terms of each arithmetic sequence.

a) 3, 7, 11, … $d = 7 - 3 = 4$

$t_4 = 11 + 4 = 15$

$t_5 = 19$

$t_6 = 23$

b) 25, 18, 11, … $d = 18 - 25 = -7$

$t_4 = 11 - 7 = 4$

$t_5 = 4 - 7 = -3$

$t_6 = -3 - 7 = -10$

c) $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots$ $d = \frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$

$t_4 = \frac{7}{4} + \frac{1}{2} = \frac{7}{4} + \frac{2}{4} = \frac{9}{4}$

$t_5 = \frac{9}{4} + \frac{2}{4} = \frac{11}{4}$

$t_6 = \frac{11}{4} + \frac{2}{4} = \frac{13}{4}$

Note that the **general arithmetic sequence** is

$$a,\ a + d,\ a + 2d,\ a + 3d,\ \ldots$$

where *a* is the first term and *d* is the common difference.

General Term of an Arithmetic Sequence

$t_n = a + (n - 1)d$, where n is a natural number

Recursive Formula of an Arithmetic Sequence

$t_1 = a$, $t_n = t_{n-1} + d$, where n is a natural number and n > 1

Ex3. Find the general term, t_n, and the recursive formula for each arithmetic sequence. Also, find the indicated term.

a) 12, 16, 20, …; t_{41} $a = 12$ $d = 4$

$t_n = 12 + (n - 1)4$

$= 12 + 4n - 4$

$\boxed{t_n = 8 + 4n}$

$t_{41} = 8 + 4(41) = 8 + 164 = 172$

Recursive Formula

$t_n = t_{n-1} + d$

$\boxed{t_n = t_{n-1} + 4 \quad t_1 = 12}$

b) -3, 2, 7, … ; t_{22} $d = 2-(-3) = 5$ $a = -3$

$t_n = a + (n-1)d$

$t_n = -3 + 5(n-1)$

$= -3 + 5n - 5$

$\boxed{t_n = 5n - 8}$ ✓

$t_{22} = -8 + 5(22)$

$= -8 + 110$

$= 102$ ✓

$t_n = t_{n-1} + d$

✓ $\boxed{t_n = t_{n-1} + 5 \quad t_1 = -3}$

Recall:

$t_1, t_2, t_3, \ldots, t_n$ n states the number of terms and

t_n is the value of the n^{th} term.

Ex4. Determine the number of terms in the finite arithmetic sequences:

a) 3, 15, 27, … , 495 $a = 3$ $d = 12$

$t_n = a + (n-1)d$

$t_n = 3 + (n-1)12$

$t_n = 3 + 12n - 12$

$t_n = 12n - 9$

$495 = 12n - 9$

$504 = 12n$

$\boxed{n = 42}$

∴ There are 42 terms in the sequence.

b) -10, -14, -18, -22, … , -138 $a = -10$ $d = -4$

$t_n = a + (n-1)d$

$t_n = -10 - 4(n-1)$

$t_n = -10 - 4n + 4$

$\boxed{t_n = -4n - 6}$

$-138 = -4n - 6$

$-132 = -4n$

$\boxed{33 = n}$

∴ There are 33 terms in the sequence.

Ex5. Given $t_{12} = 52$ and $t_2 = 102$, find *a* and *d*, then write the formula for t_n.

$t_n = a + (n-1)d$

$t_2 = a + (2-1)d$

(1) $102 = a + d$

$t_{12} = a + (12-1)d$

(2) $52 = a + 11d$

(1) $102 = a + d$

$-\quad 52 = a + 11d$

$50 = -10d$

$\boxed{-5 = d}$

$102 = a - 5$

$\boxed{107 = a}$

$t_n = a + (n-1)d$

$= 107 + (n-1)(-5)$

$t_n = 107 - 5n + 5$

$\boxed{t_n = -5n + 112}$

Ex6. A music hall has 35 seats in the front row, 44 seats in the second, 53 seats in the third, and so on.

a) How many seats are in the 12th row?

35, 44, 53 $a = 35$ $d = 9$

$t_n = a + (n-1)d$

$t_n = 35 + (n-1)9$

$t_n = 35 + 9n - 9$

$t_n = 9n + 26$

$t_{12} = 9(12) + 26$

$= 108 + 26$

$\boxed{t_{12} = 134}$

∴ There're 134 seats in the 12^{th} row.

b) If the last row has 215 seats, how many rows are there in the music hall?

35, 44, 53, . . . , 215 $a = 35$ $d = 9$ $t_n = 215$

$t_n = 9n + 26$

$215 = 9n + 26$

$189 = 9n$

$\boxed{n = 21}$

∴ There're 21 rows.