Introduction: Word Problems

Match each sentence with the correct equation (place the question number beside the answer):

1. A number **increased by** three **is equal** to nine.
$$x + 3 = 9$$

         _6_ $x - 9 = 6$

2. A number **decreased by** four **is equal** to twelve.
$$x - 4 = 12$$

         _4_ $\dfrac{x}{3} = 8$

3. Four **times** a number **is equal** to sixteen.
$$4x = 16$$

         _1_ $x + 3 = 9$

4. A number **divided by** three **is** eight.
$$\dfrac{x}{3} = 8$$

         _12_ $2x + 3y = 27$

5. John's age will be nineteen in three years.
$$x + 3 = 19$$

         _2_ $x - 4 = 12$

6. Nine years **ago**, Mary's age was six.
$$x - 9 = 6$$

         _9_ $x^2 = 16$

7. Six **less** than a number is 10.
$$x - 6 = 10$$

         _11_ $0.05x + 0.25y = 5.65$

8. A number **increased by** five **is equal** to negative four.
$$x + 5 = {}^-4$$

         _3_ $4x = 16$

9. The **square** of a number **is** sixteen.
$$x^2 = 16$$

         _7_ $x - 6 = 10$

10. A number **divided by** six **is equal** to three.
$$\dfrac{x}{6} = 3$$

         _5_ $x + 3 = 19$

11. I have x nickels and y quarters. I have **a total of** $5.65
$$0.05x + 0.25y = 5.65$$

         _10_ $\dfrac{x}{6} = 3$

12. It takes 2 hours to mow a lawn and 3 hours to weed a garden. In one month I mowed x lawns and weeded y gardens. It took 27 hours.

         _8_ $x - 5 = -4$

> **In word problems, first we change language statements into mathematical statements (equations) and then solve for the variable.**

1. Use the first clue to create a LET statement which shows how the variable represents the unknown quantities in the problem.

2. Write the second unknown in terms of the first. HINT: The second unknown is compared to the first unknown.

3. Create an equation from the second clue. Do NOT include units in equations.

4. Solve the equation.

5. Use the answer to #4 to determine the value of the other unknowns found in the LET statements.

6. Conclusion: Answer the question in sentence form – include units.

Example: Jenna ran 5 kilometres more than Kelly. The two girls ran a total of 27 kilometres. How far did each of them run?

Solution: Let "x" be the distance that Kelly ran

Kelly	Jenna
x	$x+5$

Kelly + Jenna $= 27$

$x + (x+5) = 27$

$x + x + 5 = 27$

$2x + 5 - 5 = 27 - 5$

$\dfrac{2x}{2} = \dfrac{22}{2}$

$\boxed{x = 11 \text{ km}}$

∴ Kelly ran 11 km

Jenna ran $11 + 5 = 16$ km

Number Questions

1. a) A certain number is twice another number. What are the possible values for the sum of these two numbers?

First Number	1	2	3	7	9	100	*n*
Second Number	2	4	6	14	18	200	2n
Sum	3	6	9	21	27	300	3n

b) One number is increased by 6 to get a second number. In terms of n, how can we represent:

The first number? __n__   The second number? __n+6__   The sum? __2n+6__

c) One number is decreased by 3 to get a second number. In terms of n, how can we represent:

The first number? __n__   The second number? __n−3__   The sum? __2n−3__

d) The second number is four less than seven times the first number. In terms of n, how can we represent:

$$\begin{array}{c|c} 1^{st} & 2^{nd} \\ \hline n & 7n-4 \end{array}$$

The first number? __n__   The second number? __7n−4__   The sum? __8n−4__

e) The second number is three more than one-half the first number. In terms of n, how can we represent:

$$\begin{array}{c|c} 1^{st} & 2^{nd} \\ \hline n & \frac{n}{2}+3 \end{array}$$

The first number? __n__   The second number? $\frac{n}{2}+3$ The sum? $\frac{3n}{2}+3$

Applying to Problems

2. a) One number is three times another number. If the sum of the numbers is 36, determine the two numbers.

Let "n" be the first number

$$\begin{array}{c|c} 1^{st} & 2^{nd} \\ \hline n & 3n \end{array}$$

$$n + 3n = 36$$
$$\frac{4n}{4} = \frac{36}{4}$$
$$\boxed{n = 9}$$

∴ 1st number is 9
2nd number is 3(9)=27

b) One number is four more than twice another number. If both numbers are positive and the difference between the numbers is 27, determine the numbers.

OPENING   Let "n" be the first number

1st	2nd
n	2n+4

ACTION

$$2n+4 \quad -n = 27$$

You need to subtract the 1st from the second b/c it says both # are positive.

$$n + 4 - 4 = 27 - 4$$

$$\boxed{n = 23}$$

CLOSURE ∴ 1st number is 23

2nd number is $2(23) + 4$

$$= 46 + 4$$
$$= 50$$

c) Brian ran 2 km less than Tom. They ran a total distance of 12 km. Find how far each ran.

Let "d" represent the distance Tom ran

Tom	Brian
d	d-2

$$d + d - 2 = 12$$
$$2d - 2 + 2 = 12 + 2$$
$$\frac{2d}{2} = \frac{14}{2}$$
$$\boxed{d = 7}$$

∴ Tom ran 7 km
Brian ran 5 km.

In your notebooks, give full solutions to the following problems.

3. a) Lake Ontario is 4 times as deep as Lake Erie. The sum of their depths is 300 metres. What is the depth of each lake?

   b) Angel waterfall in Venezuela is 20 times as high as Niagara Falls. The difference between their heights is 950 metres. What is the height of each of the falls?

   c) Lake Erie is 77 kilometres longer than Lake Ontario. Their length total 697 kilometres. Find the length of each lake.

   d) Lacey is three times as old as Joey. The difference in their ages is 12 years. Find their ages.

   e) Bill's mother is 22 years older than he is. The sum of their ages is 60. Find their ages.

   f) One number is 5 more than another number. Three times the greater plus twice the lesser is 30. Determine the numbers.

3)

a. Let "d" represent the depth of Lake Eerie

Eerie	Ontario
d	4d

$$d + 4d = 300$$
$$\frac{5d}{5} = \frac{300}{5}$$
$$\boxed{d = 60}$$

∴ Lake Eerie is 60m in depth
Lake Ontario is 4(60) = 240m in depth.

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b. Let "h" be the height of Niagara Falls

Opening

Niagara	Angel
h	20h

Action $20h - h = 950$
$$\frac{19h}{19} = \frac{950}{19}$$
$$h = 50m$$

Closure ∴ Niagara falls is 50m in height
Angel waterfall is 20(50) = 1000m in height

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c. Let "ℓ" be the length of Lake Ontario

Ontario	Eerie
ℓ	ℓ+77

$$\ell + \ell + 77 = 697$$
$$2\ell + 77 - 77 = 697 - 77$$
$$\frac{2\ell}{2} = \frac{620}{2}$$
$$\boxed{\ell = 310}$$

∴ Lake Ontario is 310km
Lake Eerie is 310 + 77 = 387km.

d. Let "a" represent Joey's age.

Joey	Lacy
a	$3a$

$$3a - a = 12$$
$$\frac{2a}{2} = \frac{12}{2}$$
$$\boxed{a = 6}$$

∴ Joey is 6 years old
Lacy is $3(6) = 18$ years old.

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e. Let "a" be the age of Bill.

Bill	Mother
a	$a + 22$

$$a + a + 22 = 60$$
$$2a + 22 - 22 = 60 - 22$$
$$\frac{2a}{2} = \frac{38}{2}$$
$$\boxed{a = 19}$$

∴ Bill is 19 years old
Mother is $19 + 22 = 41$ years old.

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f. Let "n" be the 1st number

1st	2nd
n	$n + 5$

$$3(n + 5) + 2n = 30$$

$$3n + 15 + 2n = 30$$
$$5n + 15 - 15 = 30 - 15$$
$$\frac{5n}{5} = \frac{15}{5}$$
$$\boxed{n = 3}$$

∴ 1st number is 3
2nd number is $3 + 5 = 8$