

Task 1: Investigating How to Solve by Graphing and Factoring

- Use the DESMOS to graph the parabola. Just provide a sketch on the paper, showing the zeros.
- Use the graph to determine the zeros.
- Factor the equation according to the type of expression (common, simple, tricky, difference of squares).

Standard Equation	Graph	Zeros/Solutions/ X-Intercepts	Factored Equation
$y = x^2 - 8x + 12$		$(2, 0)$ and $(6, 0)$ or $\{2, 6\}$	$y = (x - 2)(x - 6)$
$y = x^2 - 49$		$(-7, 0)$ and $(7, 0)$ or $\{-7, 7\}$	$y = (x + 7)(x - 7)$
$y = x^2 + 3x$		$(-3, 0)$ and $(0, 0)$ or $\{-3, 0\}$	$y = x(x + 3)$
$y = 2x^2 + 5x - 3$		$(-3, 0)$ and $(\frac{1}{2}, 0)$ or $\{-3, \frac{1}{2}\}$	$y = 2x^2 + 5x - 3$ $= 2x^2 - x + 6x - 3$ $= x(2x - 1) + 3(2x - 1)$ $= (2x - 1)(x + 3)$

What is the relationship between the zeros/solutions/x-intercepts and the factors?

When you equal the factor to zero to solve it, you find the zeros/x-int.

Hint: if the factor was $(x - 6)$, what would the corresponding zero be? $x - 6 = 0$ $x = 6$

if the factor was $(x + 4)$, what would the corresponding zero be? $x + 4 = 0$ $x = -4$

if the factor was x , what would the corresponding zero be? $x = 0$

if the factor was $(2x - 3)$, what would the corresponding zero be? $2x - 3 = 0$

$$2x = 3$$

$$x = \frac{3}{2}$$

Task 3: Solving Quadratic Equations by Factoring Practice

1. Solve each quadratic equation by factoring. Follow along the steps in the first two examples.

a. $x^2 + 7x + 12 = 0$

simple trinomial

$(x+4)(x+3) = 0$

$x+4=0 \quad x+3=0$

$x=-4 \text{ and } x=-3$

- factor the expression as appropriate (common, simple, tricky, difference of squares)
- set each factor equal to zero
- solve each of these equations for x .

b. $10x^2 + 8x = 0$

common factoring

$2x(5x+4) = 0$

$2x=0 \quad 5x+4=0$

$5x=-4$

$x=0 \text{ and } x=-\frac{4}{5}$

c. $x^2 - 8x + 16 = 0$

$$\begin{array}{c|c|c} M & A & N \\ \hline 16 & -8 & -4, -4 \end{array}$$

$(x-4)(x-4) = 0$

$x-4=0$
 $x=4$

$x-4=0$
 $x=4$

$\therefore \{4\} \text{ or } x=4$

d. $6x^2 + 9x = 0$

GCF = 3x

$3x(2x+3) = 0$

$\frac{3x}{3} = \frac{0}{3}$
 $x=0$

$2x+3=0$

$2x = -3$
 $\frac{2x}{2} = \frac{-3}{2}$

$x = -\frac{3}{2}$

$\therefore \{-\frac{3}{2}, 0\}$

g. $x^2 - 16 = 0$

$(x+4)(x-4) = 0$

$x+4=0$
 $x=-4$

$x-4=0$
 $x=4$

$\therefore \{-4, 4\}$

f. $4x^2 - 4x + 1 = 0$

$4x^2 - 2x - 2x + 1 = 0$

$2x(2x-1) - (2x-1) = 0$

$(2x-1)(2x-1) = 0$

$2x-1=0$

$x = \frac{1}{2}$

$$\begin{array}{c|c|c} M & A & N \\ \hline 4 & -4 & -2, -2 \end{array}$$

$\therefore \{\frac{1}{2}\}$

Note: Sometimes, you will need to put your equations into standard form first before you factor it.

i. $3x^2 - 2 = -7x + 4$

Collect all the terms

$3x^2 + 7x - 2 - 4 = 0$
← where the leading term is positive.

$3x^2 + 7x - 6 = 0$

Factor the tricky tri

$3x^2 - 2x + 9x - 6 = 0$

$x(3x-2) + 3(3x-2) = 0$

$(3x-2)(x+3) = 0$

$3x-2=0$

$3x=2$

$x = \frac{2}{3}$

$x+3=0$
 $x=-3$

$\therefore \{-3, \frac{2}{3}\}$

j. $4x(x+1) = 9 + 4x$

FOIL (expand)

$4x^2 + 4x = 9 + 4x$

Collect terms LS

$4x^2 + 4x - 9 - 4x = 0$

simplify

$4x^2 - 9 = 0$

Factor DS

$(2x+3)(2x-3) = 0$

$2x+3=0$

$2x=-3$

$x = -\frac{3}{2}$

$2x-3=0$

$2x=3$

$x = \frac{3}{2}$

$\therefore \{-\frac{3}{2}, \frac{3}{2}\}$

Solving by Factoring Fun!

1) $(r-4)(r+1)=0$

$$\begin{array}{l} r-4=0 \quad r+1=0 \\ r=4 \quad r=-1 \end{array} \quad \{-1, 4\}$$

3) $(b-5)(b+2)=0$

$$\begin{array}{l} b-5=0 \quad b+2=0 \\ \boxed{b=5} \quad \boxed{b=-2} \end{array} \quad \{-2, 5\}$$

5) $(p+5)(2p-1)=0$

$$\begin{array}{l} p+5=0 \quad 2p-1=0 \\ p=-5 \quad 2p=1 \\ \quad \quad 2p=1 \end{array} \quad \{-5, \frac{1}{2}\}$$

7) $n^2 - 7n + 12 = 0$ $p=1/2$

$$(n-3)(n-4)=0 \\ n=3 \quad n=4 \quad \{3, 4\}$$

9) $r^2 + 10r + 21 = 0$

$$(r+3)(r+7)=0 \\ r=-3 \text{ or } r=-7 \quad \{-7, -3\}$$

11) $m^2 + 2m = 0$

$$\begin{array}{l} m(m+2)=0 \\ m=0 \quad m+2=0 \\ \quad \quad m=-2 \end{array} \quad \{-2, 0\}$$

13) $p^2 - 6p = 0$

$$\begin{array}{l} p(p-6)=0 \\ p=0 \rightarrow p-6=0 \\ \quad \quad p=6 \end{array} \quad \{0, 6\}$$

15) $3v^2 + 3v - 60 = 0$

$$\begin{array}{l} 3(v^2 + v - 20) = 0 \\ 3(v-4)(v+5) = 0 \end{array} \quad \begin{array}{l} v-4=0 \\ \boxed{v=4} \end{array} \quad \begin{array}{l} v+5=0 \\ \boxed{v=-5} \end{array} \quad \{-5, 4\}$$

17) $5v^2 - 2v - 7 = 0$

$$\begin{array}{l} 5v^2 + 5v - 7v - 7 = 0 \\ 5v(v+1) - 7(v+1) = 0 \\ (v+1)(5v-7) = 0 \end{array} \quad \begin{array}{l} v+1=0 \\ \boxed{v=-1} \end{array} \quad \begin{array}{l} 5v-7=0 \\ 5v=7 \\ \boxed{v=7/5} \end{array} \quad \{-1, 7/5\}$$

19) $3n^2 + 11n - 4 = 0$

$$\begin{array}{l} 3n^2 - n + 12n - 4 = 0 \\ n(3n-1) + 4(3n-1) = 0 \\ (3n-1)(n+4) = 0 \end{array} \quad \begin{array}{l} 3n-1=0 \\ \boxed{n=1/3} \end{array} \quad \begin{array}{l} n+4=0 \\ \boxed{n=-4} \end{array} \quad \{-4, \frac{1}{3}\}$$

2) $(3a-2)(a+2)=0$

$$\begin{array}{l} 3a-2=0 \quad a+2=0 \\ 3a=2 \quad a=-2 \\ a=2/3 \end{array} \quad \{-2, 2/3\}$$

4) $(4n+1)(4n-5)=0$

$$\begin{array}{l} 4n+1=0 \quad 4n-5=0 \\ 4n=-1 \quad 4n=5 \\ n=-1/4 \quad n=5/4 \end{array} \quad \{-1/4, 5/4\}$$

6) $(x-5)(x-4)=0$

$$\begin{array}{l} x-5=0 \quad x-4=0 \\ x=5 \quad x=4 \end{array} \quad \{4, 5\}$$

8) $a^2 - 4a - 32 = 0$

$$(a+4)(a-8)=0 \\ a=-4 \quad a=8 \quad \{-4, 8\}$$

10) $v^2 - 3v - 10 = 0$

$$(v+2)(v-5)=0 \\ v=-2 \quad v=5 \quad \{-2, 5\}$$

12) $x^2 - 64 = 0$

$$\begin{array}{l} (x+8)(x-8)=0 \\ \boxed{x+8=0} \rightarrow \boxed{x=-8} \quad \boxed{x-8=0} \rightarrow \boxed{x=8} \end{array} \quad \{-8, 8\}$$

14) $a^2 - 4 = 0$

$$\begin{array}{l} (a+2)(a-2)=0 \\ a+2=0 \quad a-2=0 \\ a=-2 \quad a=2 \end{array} \quad \{-2, 2\}$$

16) $3r^2 - 27 = 0$

$$\begin{array}{l} 3(r^2 - 9) = 0 \\ 3(r+3)(r-3) = 0 \end{array} \quad \begin{array}{l} r+3=0 \\ r=-3 \end{array} \quad \begin{array}{l} r-3=0 \\ r=3 \end{array} \quad \{-3, 3\}$$

18) $5a^2 - 2a = 0$

$$\begin{array}{l} a(5a-2)=0 \\ \boxed{a=0} \quad 5a-2=0 \\ \quad \quad \boxed{a=2/5} \end{array} \quad \{0, 2/5\}$$

20) $5n^2 + 43n + 24 = 0$

$$\begin{array}{l} 5n^2 + 3n + 40n + 24 = 0 \\ n(5n+3) + 8(5n+3) = 0 \\ (5n+3)(n+8) = 0 \end{array} \quad \begin{array}{l} 5n+3=0 \\ \boxed{n=-3/5} \end{array} \quad \begin{array}{l} n+8=0 \\ \boxed{n=-8} \end{array} \quad \{-8, -3/5\}$$

21. $7m^2 - 13m - 24 = 0$

$7m^2 - 21m + 8m - 24 = 0$

$7m(m-3) + 8(m-3) = 0$

$(m-3)(7m+8) = 0$

$m-3=0$

$m=3$

$7m+8=0$

$m=-8/7$

$\therefore \{-8/7, 3\}$

23. $5x^2 + 12x - 9 = 0$

$5x^2 - 3x + 15x - 9 = 0$

$x(5x-3) + 3(5x-3) = 0$

$(5x-3)(x+3) = 0$

$5x-3=0$

$x=3/5$

$x+3=0$

$x=-3$

$\therefore \{-3, 3/5\}$

25. $7p^2 - 4p = 0$

$p(7p-4) = 0$

$p=0$

$7p-4=0$

$p=4/7$

$\therefore \{0, 4/7\}$

27. $-4x^2 + 5x - 15 = -5x^2 - 1$ collect terms

on 2s

$5x^2 - 4x^2 + 5x - 15 + 1 = 0$

$x^2 + 5x - 14 = 0$

$(x-2)(x+7) = 0$

$x-2=0$

$x=2$

$x+7=0$

$x=-7$

$\therefore \{-7, 2\}$

29. $3x^2 + 8x = -2x - 21 + 2x^2$

$3x^2 - 2x^2 + 8x + 2x + 21 = 0$

$x^2 + 10x + 21 = 0$

$(x+3)(x+7) = 0$

$x+3=0$

$x=-3$

$x+7=0$

$x=-7$

$\therefore \{-7, -3\}$

22. $5k^2 - 3k = 0$

GCF = k

$k(5k-3) = 0$

$k=0$

$5k-3=0$

$k=3/5$

$\therefore \{0, 3/5\}$

24. $15r^2 - 4r - 3 = 0$

$15r^2 - 9r + 5r - 3 = 0$

$3r(5r-3) + (5r-3) = 0$

$(5r-3)(3r+1) = 0$

$5r-3=0$

$r=3/5$

$3r+1=0$

$r=-1/3$

$\therefore \{-1/3, 3/5\}$

26. $8x^2 - 15x + 7 = 0$

$8x^2 - 8x - 7x + 7 = 0$

$8x(x-1) - 7(x-1) = 0$

$(x-1)(8x-7) = 0$

$x-1=0$

$x=1$

$8x-7=0$

$x=7/8$

$\therefore \{7/8, 1\}$

28. $-b^2 + 11b + 30 = -2b^2$

$2b^2 - b^2 + 11b + 30 = 0$

$b^2 + 11b + 30 = 0$

$(b+5)(b+6) = 0$

$b+5=0$

$b=-5$

$b+6=0$

$b=-6$

$\therefore \{-6, -5\}$

30. $-7a^2 - 13a + 49 = 7 - 8a^2$

$8a^2 - 7a^2 - 13a + 49 - 7 = 0$

$a^2 - 13a + 42 = 0$

$(a-6)(a-7) = 0$

$a=6$

$a=7$

$\therefore \{6, 7\}$