GEOMETRIC SEQUENCES

In a **geometric sequence,** the ratio of any term, except the first one, to the previous term is constant for all pairs of consecutive terms.
This constant or **common ratio** is denoted by ***r.***
The **first term,** t_1, is denoted by the letter *a*.

Ex1. State the common ratio, r, for each geometric sequence.

a) 4, 8, 16, 32, …

$\frac{8}{4} = 2$

$\boxed{r = 2}$

b) 100, 10, 1, 0.1, …

$\frac{10}{100} = 0.1$

$\boxed{r = 0.1}$ or $1/10$

Note that the **general arithmetic sequence** is $a, ar, ar^2, ar^3, \ldots$

where *a* is the first term and *r* is the common ratio.

General Term of a Geometric Sequence	Recursive Formula of a Geometric Sequence
$t_n = ar^{n-1}$, where *n* is a natural number	$t_1 = a$, $t_n = r\, t_{n-1}$ where *n* is a natural number and n > 1

The relationship between *n* and t_n of any geometric sequence is **nonlinear.**

Ex2. For the geometric sequence given below, find the general term and the recursive formula. Then, find t_{10}.

4, 12, 36, 108, … $a = 4$ $r = \frac{12}{4} = 3$

$t_n = ar^{n-1}$
$t_n = 4 \cdot 3^{n-1}$
$t_{10} = 4 \cdot 3^{10-1}$
$= 4 \cdot 3^9$
$\boxed{t_{10} = 78\,732}$ ✓

General Term

$\boxed{t_1 = 4, \quad t_n = 3\, t_{n-1}}$

Recursive

Ex3. Find the number of terms in the geometric sequence given below.

2, -4, 8, …, -1024 $a=2$ $r = \frac{-4}{2} = -2$

$t_n = ar^{n-1}$

$-1024 = 2(-2)^{n-1}$

$-512 = (-2)^{n-1}$

if $(-2)^9 = (-2)^{n-1}$

then $9 = n-1$

$\boxed{n=10}$

∴ There're 10 terms.

Ex4. For a geometric sequence, $t_2 = 8$ and $t_5 = 64$. Find *a, r,* and t_n.

$\underline{t_1}$, 8, —, —, 64, ……, $\underline{t_n}$

t_2 t_5

$t_n = a(r)^{n-1}$

$t_2 = a(r)^{2-1}$

$8 = a \cdot r$

$\star\ \frac{8}{r} = a$

$t_5 = a(r)^{5-1}$

$64 = ar^4$

sub "$\frac{8}{r}$" for a

$64 = \frac{8}{r} \cdot r^4$

$64 = 8r^{4-1}$

$8 = r^3$

if $2^3 = r^3$ then $\boxed{r=2}$

$\frac{8}{2} = a$

$\boxed{a=4}$

$t_n = 4(2)^{n-1}$

Ex5. In a laboratory, a bacterial population doubles every hour. At 1 pm, the population is 20000 cells. How many cells will be present at 10 pm?

1pm	2pm		10pm
20,000	40,000	. . . .	x

$a = 20000$

$r = 2$

$t_n = ar^{n-1}$

$t_n = 20000(2)^{n-1}$

$t_{10} = 20000(2)^9$

$= 10,240,000$

∴ At 10pm there'll be 10,240,000 bacteria.

Ex6. A store has a sale in which 10% is taken off of the cost of an item at the end of each day. Suppose the item originally costs $250.

a) Determine its cost (at the end of the day) for each of the next five days.

$a = 225 \quad r = 0.9 \qquad t_n = ar^{n-1}$

$\underset{\text{Day 1}}{\underline{225}}, \underset{2}{\underline{202.5}}, \underset{3}{\underline{182.25}}, \underset{4}{\underline{164.025}}, \underset{5}{\underline{147.62}}$

Day1 $= 250 \times 90\%$
$= 225$

Day2 $= 225 \times 90\%$
$= 202.5$

Day3 $= 202.5 \times 90\%$
$= 182.25$

Day4 $= 182.25 \times 90\%$
$= 164.025$

Day5 $= 164.025 \times 90\%$
$= 147.62$

b) About how many days will it take for the sale price to be less than $90?

$225, \ 202.5, \ 182.25, \ 164.03, \ 147.62 \qquad a = 225 \quad r = 0.9$

$t_n = a(r)^{n-1}$

$t_n = 225(0.9)^{n-1}$

$90 > 225(0.9)^{n-1} \rightarrow$ less than $\$90$

$0.4 > (0.9)^{n-1} \longrightarrow$ trial and error method (no log)

$(0.9)^8 = 0.43$

$\boxed{(0.9)^9 = 0.39}$

$n - 1 \doteq 9$

$n \doteq 10$ $\quad \therefore$ By the 10th day, it'll be less than $90

$t_{10} = 225(0.9)^{10-1}$

$= \$87.16$