OPERATIONS WITH FRACTIONS

A fraction is made up of two parts:
The top of the fraction is called the numerator and the bottom of the fraction is called the denominator.

If the numerator is greater than the denominator, the fraction is called an **improper fraction**. (Ex: $\frac{8}{3}$). These types

of fractions can also be written as a whole number and a fraction. This is called a **mixed number** (Example: $2\frac{2}{3}$).

REDUCING FRACTIONS

When dealing with fractions, we need to express them in the most reduced form.

Ex: $\frac{3}{6}$ is not in its most reduced form because 3 and 6 are both divisible by 3. As soon as you find a number that will divide the numerator and denominator, you have to reduce them.

METHOD 1	METHOD 2: CALCULATOR
$\frac{3}{6} = \frac{3 \div 3}{6 \div 3}$ $= \frac{1}{2}$	Input the numbers into your calculator as follows: 3 \[A b/c\] 6 $= \frac{1}{2}$

Reduce the following fractions without a calculator, then check the accuracy of your answer with a calculator:

a. 4/16	b. 14/35	c. $2\frac{4}{12}$	d. 30/ 50
$= \frac{4 \div 4}{16 \div 4} = \frac{1}{4}$	$= \frac{14 \div 7}{35 \div 7} = \frac{2}{5}$	$= 2\frac{4 \div 4}{12 \div 4} = 2\frac{1}{3}$	$= \frac{30 \div 10}{50 \div 10} = \frac{3}{5}$

CONVERTING MIXED NUMBERS INTO IMPROPER FRACTIONS

To convert mixed numbers to improper fractions:

$$w\frac{n}{d} = \frac{w \times d + n}{d}$$

METHOD 1: PENCIL AND PAPER	METHOD 2: CALCULATOR
$2\frac{3}{5} = \frac{2 \times 5 + 3}{5} = \frac{10 + 3}{5} = \frac{13}{5}$ If $2\frac{3}{5} = 13/5$ Then $-2\frac{3}{5} = -13/5$	2 \[A b/c\] 3 \[A b/c\] 5 $= 2 u 3/5$ \[2nd\] \[A b/c\] \[=\] $\frac{13}{5}$

Try these:

a. $4\frac{7}{8} = \frac{4 \times 8 + 7}{8}$
$= \frac{32 + 7}{8}$
$= \frac{39}{8}$

b. $3\frac{1}{2} = \frac{3 \times 2 + 1}{2}$
$= \frac{6 + 1}{2}$
$= \frac{7}{2}$

c. $5\frac{11}{12} = \frac{5 \times 12 + 11}{12}$
$= \frac{60 + 11}{12}$
$= \frac{71}{12}$

d. $-2\frac{4}{7} = -\left(\frac{2 \times 7 + 4}{7}\right)$
$= -\left(\frac{14 + 4}{7}\right)$
$= -\frac{18}{7}$

e. $-3\frac{1}{5} = -\left(\frac{3 \times 2 + 1}{5}\right)$
$= -\left(\frac{6 + 1}{5}\right)$
$= -\frac{7}{5}$

ADDING AND SUBTRACTING FRACTIONS

To add or subtract fractions the <u>denominators</u> must be the same. After finding the lowest (least) common denominator, you can add or subtract the numerators, and the denominators stay the same.

Ex: $1/3 + 1/6 = ?$

Step 1: List multiples of each denominator	Step 2: Circle the smallest common number of each set of multiples
multiples of 3 = **3, 6, 9, 12** multiples of 6 = **6, 12, 18, 24**	The answer is __**6**__. This will be your lowest common denominator.
Step 3: Multiply each fraction (top and bottom numbers) by the number to make each denominator 6. $= \dfrac{1 \times 2}{3 \times 2} + \dfrac{1 \times 1}{6 \times 1}$	**Step 4: Write the numerators (top numbers) over your lowest common denominator and simplify (combine numbers). Reduce your final answer.** $= \dfrac{2+1}{6}$ $= \dfrac{3}{6}$ $= \dfrac{1}{2}$

Try these:

a. $\dfrac{-1}{2} + \dfrac{3}{4}$ $= \dfrac{-1\times 2}{2\times 2} + \dfrac{3}{4}$ $= \dfrac{-2}{4} + \dfrac{3}{4}$ $= \dfrac{-2+3}{4}$ $= \dfrac{1}{4}$ ① Find the lowest common multiple $2 \to ④ \to 8 \to 16 \to \cdots$ $④ \to 8 \to 16 \to \cdots$ LCM = 4 ② multiply first fraction by 2	**b.** $\dfrac{5}{6} - \left(\dfrac{3}{4}\right)$ $= \dfrac{5\times 2}{6\times 2} - \dfrac{3\times 3}{4\times 3}$ $= \dfrac{10}{12} - \dfrac{9}{12}$ $= \dfrac{10-9}{12}$ $= 1/12$ $\times 2$ $6 - ⑫ - 18 - 24$ $4 - 8 - ⑫ - 16$ $\times 3$
c. $1\dfrac{1}{2} + 2\dfrac{3}{4}$ *convert to improper first* $= \dfrac{2\cdot 3}{2\cdot 2} + \dfrac{11}{4}$ LCD = 4 $= \dfrac{6+11}{4}$ $= \dfrac{17}{4}$ or $4\dfrac{1}{4}$	**d:** $4\dfrac{1}{5} - 2\dfrac{2}{3}$ *convert to improper first* $= \dfrac{4\times 5+1}{5} - \dfrac{2\times 3+2}{3}$ $= \dfrac{21\cdot 3}{5\cdot 3} - \dfrac{8\cdot 5}{3\cdot 5}$ $= \dfrac{63-40}{15}$ $= \dfrac{23}{15}$ OR $= 1\dfrac{8}{15}$

MULTIPLYING FRACTIONS

To multiply fractions, simply multiply the numerators and multiply the denominators.

Ex1: $\dfrac{1}{7} \times \dfrac{2}{3}$ $= \dfrac{1 \times 2}{7 \times 3}$ $= \dfrac{2}{21}$	**Ex2**: $\dfrac{1}{4} \times \dfrac{8}{3}$ $= \dfrac{1 \times 8}{4 \times 3}$ $1\ \boxed{A^b/_c}\ 4 \times 8\ \boxed{A^b/_c}\ 3$ $= \dfrac{8 \div 4}{12 \div 4}$ OR $= 2/3$ $= \dfrac{2}{3}$
Ex3: $2\dfrac{2}{5} \times 3\dfrac{1}{4}$ * convert to improper fraction $= \dfrac{2 \times 5 + 2}{5} \cdot \dfrac{3 \times 4 + 1}{4}$ $= \dfrac{10 + 2}{5} \cdot \dfrac{12 + 1}{4}$ $= \dfrac{\overset{3}{\cancel{12}}}{5} \cdot \dfrac{13}{\underset{1}{\cancel{4}}}$ $= 39/5$	**Ex4**: $3\dfrac{1}{5} \times 2$ * convert to improper fraction You can express 2 as a fraction by putting 2 over 1. $= \dfrac{3 \times 5 + 1}{5} \cdot 2$ $= \dfrac{15 + 1}{5} \cdot 2$ $= \dfrac{16 \times 2}{5 \times 1}$ $= \dfrac{16}{5} \cdot \dfrac{2}{1}$ $= 32/5$

DIVIDING FRACTIONS

To divide fractions, flip the second fraction upside down then multiply the fractions as above.

a. $\dfrac{1}{2} \div \dfrac{8}{3}$ $= \dfrac{1}{2} \times \dfrac{3}{8}$ $= \dfrac{1 \times 3}{2 \times 8}$ $= \dfrac{3}{16}$	**b.** $3 \div \dfrac{1}{4}$ *write 3 over 1 $= 3 \times \dfrac{4}{1}$ $= 12$
c. $2\dfrac{1}{2} \div 3\dfrac{5}{4}$ *convert to improper fraction $= \dfrac{2 \times 2 + 1}{2} \div \dfrac{3 \times 4 + 5}{4}$ $= \dfrac{5}{2} \div \dfrac{17}{4}$ $= \dfrac{5}{\cancel{2}} \times \dfrac{\overset{2}{\cancel{4}}}{17}$ 2 divides 4 into 2 $= 10/17$	**d.** $3\dfrac{2}{5} \div 2\dfrac{1}{4}$ *convert to improper fraction $= \dfrac{3 \times 5 + 2}{5} \div \dfrac{2 \times 4 + 1}{4}$ $= \dfrac{17}{5} \div \dfrac{9}{4}$ $= \dfrac{17}{5} \times \dfrac{4}{9}$ $= \dfrac{68}{45}$

PRACTICAL PROBLEMS

1. If an electrician works 4 3/4 hours a day on a job at $16 per hour, how much money does she earn in 5 days?

1 day
$= 16 \times 4\frac{3}{4}$

5 days
$= 76 \times 5$

$= 16 \times \frac{19}{4}$

$= 380$

$= 76$

∴ They make $380 in five days.

2. A department in a factory has three 3/4 - horsepower motors, five 1/4-horsepower motors, six 3 1/3 - horsepower motors and eight 7 1/2 horsepower motors. What is the total connected motor load in horsepower rating?
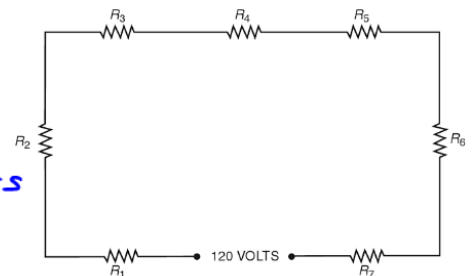
$$= 3 \times \frac{3}{4} + 5 \times \frac{1}{4} + 6 \times 3\frac{1}{3} + 8 \times 7\frac{1}{2} \rightarrow = \frac{14}{4} + 20 + 60$$

$$= \frac{9}{4} + \frac{5}{4} + \frac{60}{3} + \frac{120}{2} \qquad = 80 + \frac{7}{2}$$

$$= 80 + 3\frac{1}{2}$$

∴ Total connected motor load is 83½ horsepower.

3. If 12 6/7 watts are distributed equally over each of the resistors shown in this figure, find the average number of watts per resistor.

$= 12\frac{6}{7} \div 7$

$= 1\frac{41}{49}$

∴ The average number of watts per resistor is $1\frac{41}{49}$ watts.

4. If 7 1/3 yards of varnished cambric (insulation) cost $22, what is its cost per yard?

$= 22 \div 7\frac{1}{3}$

$= 22 \div \frac{7 \times 3 + 1}{3}$

$= 22 \div \frac{22}{3}$

$= 22 \times \frac{3}{22}$

$= 3$

∴ It costs $3/yard.