Date: _____

DETERMINING ANGLES IN RIGHT TRIANGLES

RECALL the three primary trigonometry ratios.

$\sin\theta = \frac{\text{OPPOSITE}}{\text{HYPOTENUSE}}$	$\cos\theta = \frac{\text{ADJACENT}}{\text{HYPOTENUSE}}$	$\tan\theta = \frac{\text{OPPOSITE}}{\text{ADJACENT}}$

For every trigonometry ratio there is an **INVERSE** ratio. It is used to calculate **ANGLES**.

Inverse ratios are usually found on a scientific calculator by using the 2nd F, INV, or SHIFT key

The inverse for **sin** is: $\sin^{-1}$ The inverse for **cos** is: $\cos^{-1}$ The inverse for **tan** is: $\tan^{-1}$

KEY WORDS
Opposite/Hypotenuse
Adjacent/Hypotenuse
Opposite/Adjacent
Inverse
$\sin^{-1}$
$\cos^{-1}$
$\tan^{-1}$
acute
measure
two

CASE A) DETERMINING THE ANGLE WITH INVERSE RATIO

Solved Example: Calculate the angles given. Round your answer to whole degree.

$\sin\theta = 0.667$ *swap θ *with 0.667*
$\sin^{-1} 0.667 = \theta$ *sin becomes $\sin^{-1}$
$\theta = 4$

Example: Calculate each of the angles given. Round your answer to whole degree.

a) $\cos\theta = 0.667$

$\cos^{-1} 0.667 = \theta$
$\theta \approx 48$

b) $\tan\theta = 0.667$

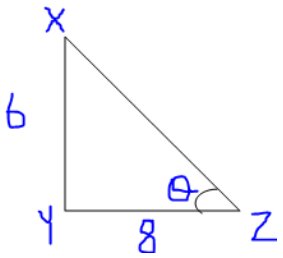
$\tan^{-1} 0.667 = \theta$
$\theta = 34$

CASE B) DETERMINING THE ANGLE FROM THE TRIANGLE

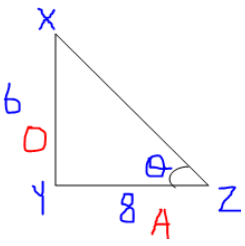
To find the measure of a(n) acute angle in a right-angle triangle, it is necessary to have the measure of any two sides of the triangle.

3 Steps to Solving ANGLES
Step 1: Label the sides of your triangle relative to the angle you want to find
Step 2: Determine which trig ratio to use (sin, cos, tan)
Step 3: Set up the equation with the unknown and solve using the ***inverse*** trig ratio ($\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$).

Solved Example: Determine the angle shown to the nearest degree.



Step 1:



Step 2: with sides O and A we calculate tan ratio

Step 3: $tan\,\theta = \frac{6}{8}$ *swap 6/8 with θ

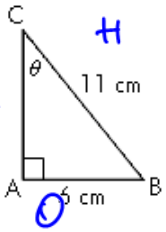
$\tan^{-1}\frac{6}{8} = \theta$

$\therefore$ Angle θ is approximately 37°.

PRACTICE

Example: Find each of the angles shown, rounded to one decimal place.

a) Find $\angle C$



SOH H O

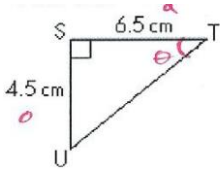
$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{6}{11}$

$\sin^{-1}\left(\frac{6}{11}\right) = \theta$

$\theta \doteq 33.1°$

b) Find $\angle T$



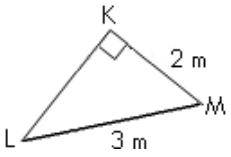
$\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{4.5}{6.5}$

$\tan^{-1}\left(\frac{4.5}{6.5}\right) = \theta$

$\theta \doteq 34.7°$

c) Find $\angle M$



$\cos\alpha = \frac{A}{H}$

$\cos\alpha = \frac{2}{3}$

$\cos^{-1}\left(\frac{2}{3}\right) = \alpha$

$\alpha \doteq 48.2°$

1. Evaluate each of the following to the nearest degree.

a) $\sin a = 0.34$ $\sin^{-1} 0.34 = a \Rightarrow a \doteq 20°$

b) $\cos b = 0.5$ $\cos^{-1} 0.5 = b$ $b = 60°$

c) $\tan c = 0.466$ $\tan^{-1} 0.466 = c$ $c \doteq 25°$

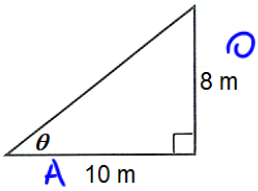
d) $\sin d = 0.951$ $\sin^{-1} 0.951 = d$ $d \doteq 72°$

e) $\cos e = 0.574$ $\cos^{-1} 0.574 = e$ $e \doteq 55°$

f) $\tan f = 0.268$ $\tan^{-1} 0.268 = f$ $f \doteq 15°$

2. Find each of the angles shown. Round to one decimal.

a)



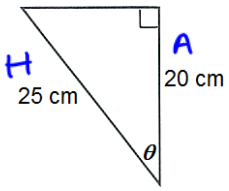
O

$\tan\theta = \frac{8}{10}$

$\tan^{-1}\left(\frac{8}{10}\right) = \theta$

$\theta \doteq 38.7$

b)

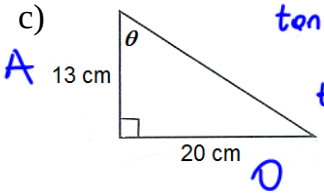


$\cos\theta = \frac{20}{25}$

$\cos^{-1}\left(\frac{20}{25}\right) = \theta$

$\theta \doteq 36.9°$

c)

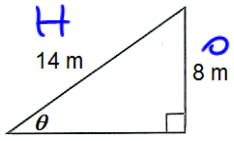


$\tan\theta = \frac{20}{13}$

$\tan^{-1}\left(\frac{20}{13}\right) = \theta$

$\theta \doteq 57°$

d)

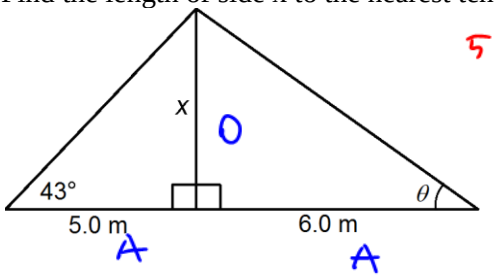


$\sin\theta = \frac{8}{14}$

$\sin^{-1}\left(\frac{8}{14}\right) = \theta$

$\theta \doteq 34.8°$

3. Find the length of side x to the nearest tenth of a metre and of angle θ to the nearest degree.



$5 \times \tan 43° = \frac{x}{5} \times 5$

$x = 5 \times \tan 43$

$x \doteq 4.7\text{m}$

$\tan\theta = \frac{x}{6}$

$\tan\theta = \frac{4.7}{6}$

$\tan^{-1}\left(\frac{4.7}{6}\right) = \theta$

$\theta \doteq 38°$

4. Based on the following diagram use the values given to find the missing angles indicated.

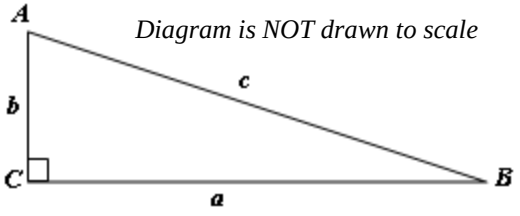
a) a = 55 m, b = 137 m → find $\angle A$, $\angle B$

b) a = 235 cm, c = 268 cm → find $\angle A$, $\angle B$

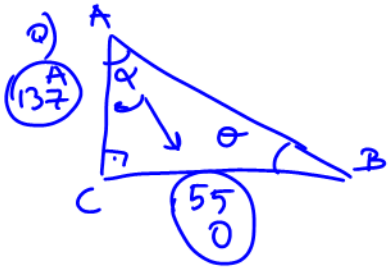
c) b = 21 mm, c = 40 mm → find $\angle A$, $\angle B$

d) a = 30 cm, b = 285 cm → find $\angle A$, $\angle B$

Diagram is NOT drawn to scale



a)



$\alpha + \theta + 90 = 180$

$\alpha + \theta = 90°$

$\tan\alpha = \frac{55}{137}$ $\Rightarrow \tan^{-1}\left(\frac{55}{137}\right) = \alpha$ $\Rightarrow \boxed{\alpha \doteq 22°}$

$\alpha + \theta = 90$ $\quad \alpha = 22$

$22 + \theta = 90$

$\theta = 90 - 22$

$\boxed{\theta = 68°}$

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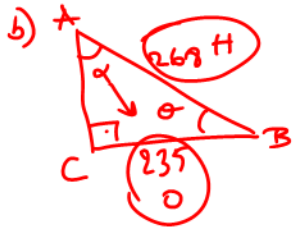
b)



$\sin\alpha = \frac{235}{268}$ $\Rightarrow \sin^{-1}\left(\frac{235}{268}\right) = \alpha$ $\Rightarrow \boxed{\alpha \doteq 61}$

$\alpha + \theta = 90$

$61 + \theta = 90$

$\theta = 90 - 61$

$\boxed{\theta = 29°}$

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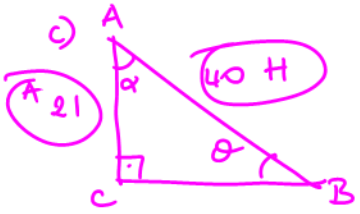
c)



$\cos\alpha = \frac{21}{40}$ $\Rightarrow \cos^{-1}\left(\frac{21}{40}\right) = \alpha$ $\Rightarrow \boxed{\alpha \doteq 58°}$

$\alpha + \theta = 90°$

$58 + \theta = 90$

$\theta = 90 - 58$

$\boxed{\theta = 32°}$

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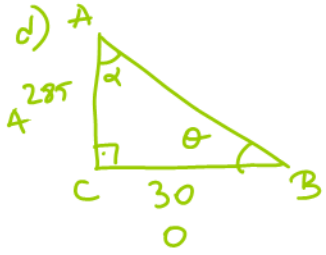
d)



$\tan\alpha = \frac{30}{285}$

$\tan^{-1}\left(\frac{30}{285}\right) = \alpha$

$\boxed{\alpha = 6°}$

$\alpha + \theta = 90$

$6 + \theta = 90$

$\boxed{\theta = 84°}$