RATE OF CHANGE (ROC)

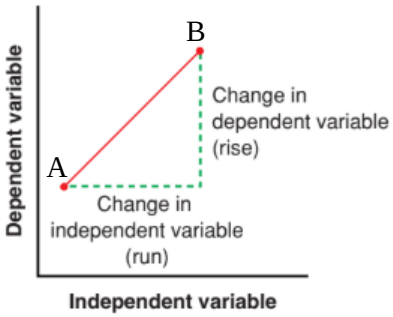
The rate at which something is changing. The ROC is often indicated by the **slope** of a graph.

We can determine an average rate of change using a table or graph

Table:

	Independent Variable (x)	Dependent Variable (y)
POINT A →	x_1	y_1
POINT B →	x_2	y_2

Graph:



Average ROC (slope) = $\frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$

CALCULATING AND INTERPRETING RATES OF CHANGE

EXAMPLE 1: Calculate the average rate of change between each pair of points. Explain what the rate of change means.

a) Altitude of an Airplane

Time (min)	Airplane Height (m)
x_1 0	y_1 2000
x_2 4	y_2 1400

$ROC = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{1400 - 2000}{4 - 0}$

$= \frac{-600}{4}$

$= -150$

∴ Every min the plane's altitude drops by 150m.

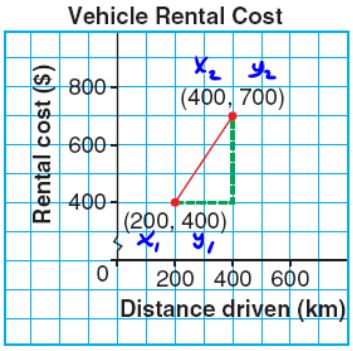
b)

$ROC = \frac{730 - 400}{400 - 200}$

$= \frac{300}{200}$

$= 1.5$

∴ For every km driven you pay $1.50.



EXAMPLE 2: The graph below represents the relationship between the number of hours worked and the weekly salary an employee earns.

Calculate both rates of pay from the graph.

$ROC(Regular) = \frac{400 - 0}{40 - 0}$

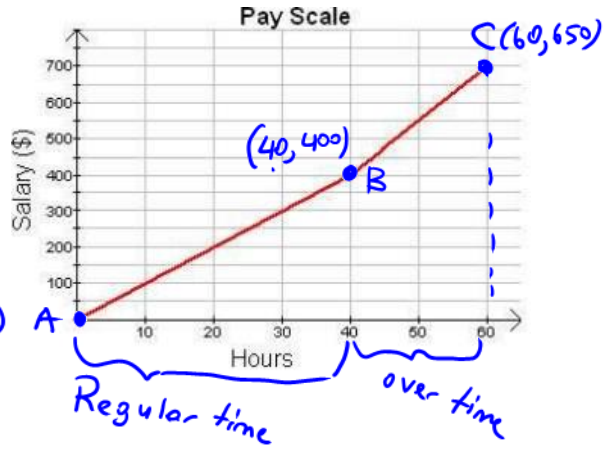
$= \frac{400}{40}$

$= \$10/1h$

$ROC(O.T) = \frac{650 - 400}{60 - 40}$

$= \frac{250}{20}$

$= \$12.5/1h$



COMPARING RATES OF CHANGE

EXAMPLE 2: The distance required to stop a car depends on the speed at which the car is travelling. Use the tables below (showing the reaction distance and breaking distance needed to stop a car on dry pavement for given speed) to answer the following.

a) Calculate the average rate of change between consecutive points in each table. Describe the rates of change revealed in each table.

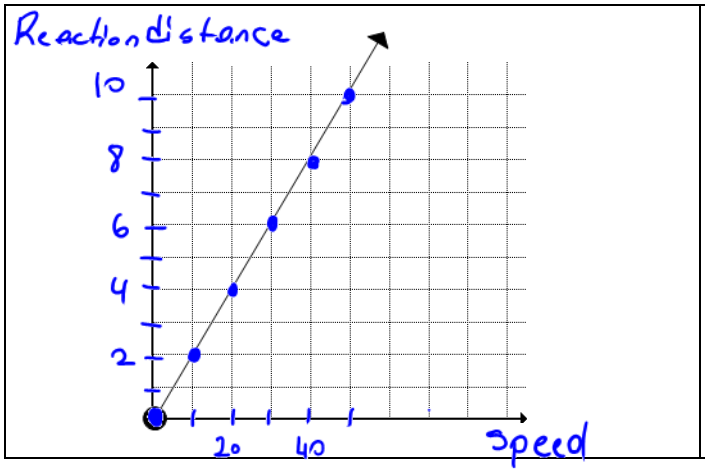
Speed (km/h)	Reaction Distance (m)	$\frac{change\ in\ distance}{change\ in\ speed}$
0	0	$\frac{2-0}{10-0}=\frac{2}{10}=0.2$
10	2	$\frac{4-2}{20-10}=\frac{2}{10}=0.2$
20	4	$\frac{6-4}{30-20}=\frac{2}{10}=0.2$
30	6	$\frac{8-6}{40-30}=\frac{2}{10}=0.2$
40	8	$\frac{10-8}{50-40}=\frac{2}{10}=0.2$
50	10	

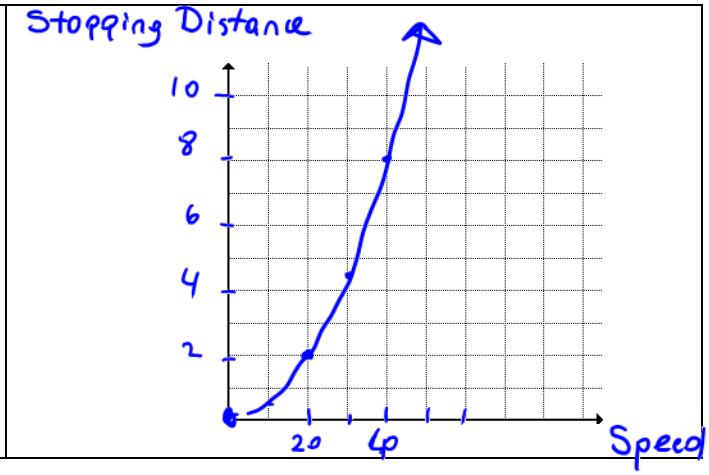
The rates of change are constant at 0.2.
The reaction distance increased by 0.2 for 1 km/h

Speed (km/h)	Stopping Distance (m)	$\frac{change\ in\ distance}{change\ in\ speed}$
0	0.0	$\frac{0.5-0}{10-0}=\frac{0.5}{10}=0.05$
10	0.5	$\frac{2-0.5}{20-10}=\frac{1.5}{10}=0.15$
20	2.0	$\frac{4.5-2}{30-20}=\frac{2.5}{10}=0.25$
30	4.5	$\frac{8-4.5}{40-30}=\frac{3.5}{10}=0.35$
40	8.0	$\frac{12.5-8}{50-40}=\frac{4.5}{10}=0.45$
50	12.5	

The rates of change are **increasing**.
NOT CO

b) Graph the data in the tables. Describe how the graph reflects the rates of change across the data.



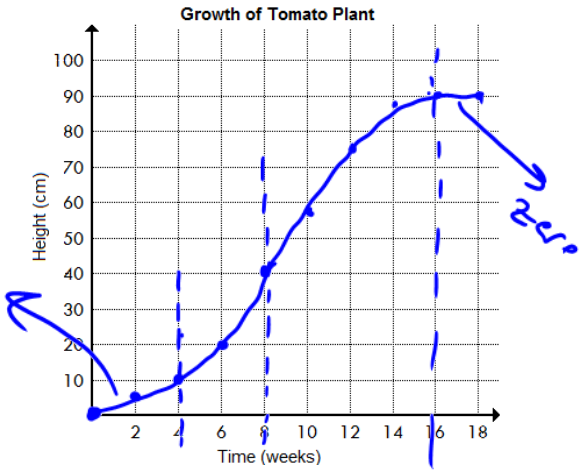


- First differences (ROC) are the same.
LINEAR

ROC not the same.
NON-LINEAR

EXAMPLE 3 The table below shows the change in height of a tomato plant from germination until the tomato ripens.

Time (wks)	Height (cm)
0	0
2	5
4	10
6	20
8	40
10	58
12	75
14	86
16	90
18	90



a) Determine when the **rate of change** (slope) in the height is:

i) **Zero (calculate 1st differences)**: between weeks 16 and 18

ii) **Constant**: between 0 and 4

iii) **Changing**: 4 and 16

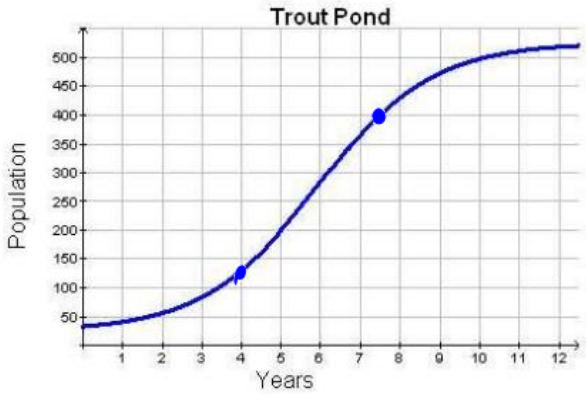
b) When is the rate of change in height the greatest?

between 6 and 8 because height grew 20 cm in 2 weeks.

c) Describe the growth of the plant.

Between weeks 0 and 4, it grows at a constant rate. Between 4 and 6 it grows rapidly with a changing ROC. Between weeks of 8 and 16, the speed of growth slows down and it stops on week 16.

EXAMPLE 4: Conservationists introduce a small number of trout into a pond. The graph below illustrates the population growth over many years.



a) Is the rate of change of the population constant? Explain how you reasoning.

No. Line is not straight.

b) When is the population growing the slowest and fastest? Why might the population growth follow this pattern?

Slowest between years 0 and 4

Fastest 4 and 7.5

At first there was enough resources for the fish to thrive however as the population reached 400, growth slows down and caps around 500.

***SUMMARY* (*Fill the table below using your textbook page 283*)**

Rate of Change	Table	Example on Graph
Zero	First differences are Zero	
Constant	First differences are the same	
Changing	First differences are changing