Evaluate each of the following.

a. $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^{\frac{1}{2}+\frac{1}{2}} = 9$

b. $\sqrt{9} \times \sqrt{9} = 9$

Therefore, $9^{\frac{1}{2}} = \sqrt{9}$

In general $a^{\frac{1}{2}} = \sqrt{a}$

c. $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 8$

d. $\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = 2 \times 2 \times 2$
$= 8$

Therefore: $8^{\frac{1}{3}} = \sqrt[3]{8}$

In general: $b^{\frac{1}{3}} = \sqrt[3]{b}$

Exponents in the form $\frac{1}{n}$

radical sign

index

$x^{\frac{1}{n}} = \sqrt[n]{x}$ where n is a natural number. (Read the n^th root of x)

radicand

Examples: Write each of the following in radical form. Evaluate, if possible.

a. $64^{\frac{1}{2}}$
$= \sqrt{64}$
$= \sqrt{8^2}$
$= 8$

b. $27^{\frac{1}{3}}$
$= \sqrt[3]{27}$
$= \sqrt[3]{3^3}$
$= 3^{3/3}$
$= 3$

c. $1024^{\frac{1}{5}}$
$= \sqrt[5]{1024}$
$= \sqrt[5]{4^5}$
$= 4$

d. $16^{\frac{1}{4}}$
$= \sqrt[4]{16}$
$= 2$

e. $9^{-\frac{1}{2}}$
$\frac{1}{9^{1/2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

f. $(-64)^{\frac{1}{3}}$
$= \sqrt[3]{-64}$
$= -4$

g. $(-243)^{\frac{1}{5}}$
$= \sqrt[5]{-243}$
$= -3$

h. $(-625)^{\frac{1}{4}}$
not possible to evaluate

i. $(-81)^{\frac{1}{2}}$
not possible to evaluate

Take Note:

Given $\sqrt[n]{x}$, if n is an even number, then $x \geq 0$ for the n^th root to be real.
if n is an odd number, then x can be any real number.

Use the exponent laws to express $x^{\frac{2}{3}}$ in two ways. **Recall:** Power law $(a^m)^n = a^{mn}$

a. $x^{\frac{2}{3}}$ $= (x^2)^{1/3} = \sqrt[3]{x^2}$

b. $x^{\frac{2}{3}}$ $= x^{\frac{1}{3}} \cdot x^{\frac{1}{3}}$

$= \sqrt[3]{x} \cdot \sqrt[3]{x} = (\sqrt[3]{x})^2$

Exponents in the form $\frac{m}{n}$:

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$ where m and n are natural numbers.

Examples: Write each of the following in radical form. Evaluate, if possible.

a. $8^{\frac{2}{3}}$ $= \sqrt[3]{8^2}$ $= (\sqrt[3]{8})^2$

$= \sqrt[3]{64}$ or $= 2^2$

$= 4$ $= 4$

b. $-25^{\frac{5}{2}}$ $= -\sqrt{25^5}$ or $-(\sqrt{25})^5$

$= -\sqrt{5^{10}}$ $= -5^5$

$= -5^5$ $= -3125$

c. $256^{-\frac{3}{4}}$ $= \frac{1}{256^{3/4}} = \frac{1}{(\sqrt[4]{256})^3}$

$= \frac{1}{4^3} = \frac{1}{64}$

d. $64^{-1.5}$

$64^{-3/2} = \frac{1}{64^{3/2}} = \frac{1}{(\sqrt{64})^3}$

$= \frac{1}{8^3} = 1/512$

e. $(-27^{-\frac{2}{3}})$

$\frac{1}{(-27^{2/3})} = \frac{1}{(\sqrt[3]{-27})^2} = \frac{1}{(-3)^2} = \frac{1}{9}$

f. $-27^{-\frac{2}{3}}$

$-\frac{1}{27^{2/3}} = -\frac{1}{(\sqrt[3]{27})^2} = -\frac{1}{(3)^2} = -\frac{1}{9}$

g. $\sqrt[2]{\sqrt{x^{12}}}$

$= \sqrt{x^{12/2}}$

$= \sqrt{x^6}$ $= x^{6/2}$

$= x^3$

h. $\sqrt{\sqrt[3]{y^4}}$ $= \sqrt{y^{4/3}}$

$= y^{\frac{4}{3} \div 2}$ $= y^{\frac{4}{3} \times \frac{1}{2}}$

$= y^{2/3}$

$= \sqrt[3]{y^2}$

i. $\sqrt{81m^8}$ $= \sqrt{3^4 \cdot m^8}$

$= 3^{4/2} \cdot m^{8/2}$

$= 3^2 \cdot m^4$ $\therefore 9m^4$

j. $\sqrt{\sqrt{10000y^6}}$ $= \sqrt{10^{4/2}\, y^{6/2}}$

$= \sqrt{10^2 y^3}$

$= 10^{2/2}\, y^{3/2}$

$= 10\sqrt{y^3}$

k. $\sqrt{\sqrt[3]{64x^{18}}}$

$= \sqrt{2^{6/3}\, x^{18/3}}$

$= \sqrt{2^2 x^6}$

$= 2^{2/2}\, x^{6/2}$

$= 2x^3$

l. $\sqrt[5]{\frac{\sqrt{x}\sqrt{x^3}}{x^{\frac{3}{4}}}}$ $= \sqrt[5]{\frac{\sqrt{x \cdot x^3}}{x^{3/4}}}$

$= \sqrt[5]{\frac{\sqrt{x^4}}{x^{3/4}}}$

$= \sqrt[5]{\frac{x^2}{x^{3/4}}}$

$= \sqrt[5]{x^{2-3/4}}$ $\frac{8}{4} - \frac{3}{4}$

$= \sqrt[5]{x^{5/4}}$

$= x^{\frac{5}{4} \div 5}$

$= x^{\frac{5}{4} \times \frac{1}{5}}$

$= x^{\frac{1}{4}}$ $\sqrt[4]{x}$

m. $\left(\frac{-8x^3}{216}\right)^{-\frac{1}{3}}$

$= \left(\frac{216}{-8x^3}\right)^{1/3}$

$= \left(\frac{6^3}{-2^3x^3}\right)^{1/3}$

$= \frac{6^{3 \cdot \frac{1}{3}}}{(-2^3)^{1/3} \cdot (x^3)^{1/3}}$

$= \frac{6}{-2 \cdot x}$

$= \frac{-3}{x}$

n. $\left(\frac{16}{81y^8}\right)^{-\frac{3}{4}}$

$= \left(\frac{81y^8}{16}\right)^{3/4}$

$= \left[\frac{(3^4)y^8}{2^4}\right]^{3/4}$

$= \frac{3^{4 \cdot \frac{3}{4}} \cdot y^{8 \cdot \frac{3}{4}}}{2^{4 \cdot \frac{3}{4}}}$

$= \frac{3^3 y^6}{2^3} = \frac{27y^6}{8}$