

Theoretical Probability

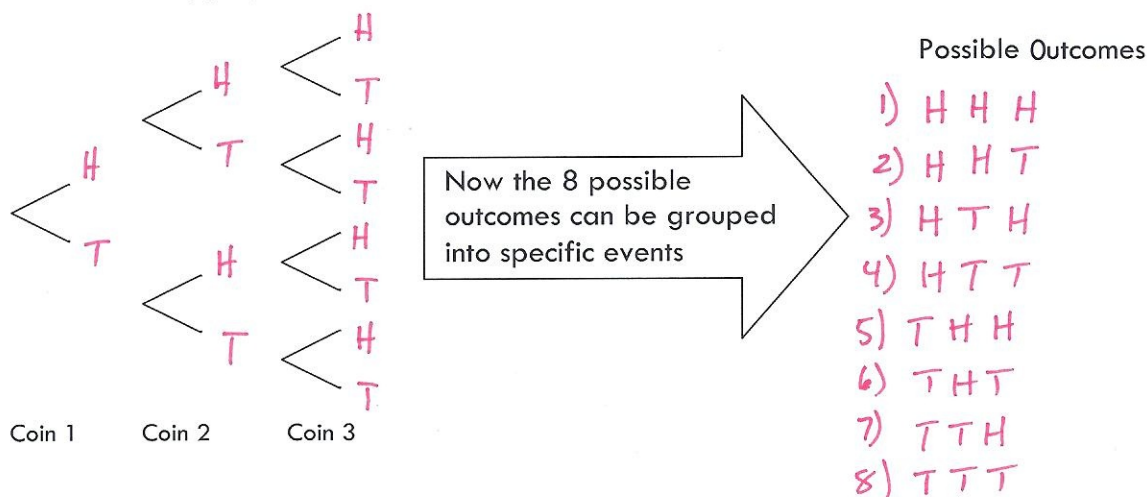
Theoretical Probability is another measure of the likelihood of an event. Without performing experiments, theoretical probability measures possible outcomes

Theoretical Probability Formula:
$$P(\text{event}) = \frac{\text{\# of successful outcomes}}{\text{total \# possible outcomes}}$$

To calculate theoretical probability, all outcomes must be equally likely, meaning each event has the same chance of occurring because the conditions are fair.

Again, the probability of an event is a value between 0 and 1.

Recall the coin flipping experiment. Use a tree diagram to help determine the possible outcomes.



Event A (earning 3 points) is outcome # 8 (3 tails)

Event B (earning 1 point) is outcome # 4, 6, 7 (2 tails)

Event C (earning 0 points) is outcome # 1, 2, 3, 5 (0 or 1 tail)

Listing the outcomes and the events gives a more clear indication of the chances of having a particular event occur. In this case you can calculate the theoretical probability for each of the events.

Probability for Event A

$$P(A) = \frac{\text{\# successful outcome}}{\text{total possible outcomes}}$$

$$= \frac{1}{8}$$

$$= 0.125$$

$$= 12.5\%$$

Probability for Event B

$$P(B) = \frac{\text{\# successes}}{\text{\# possibilities}}$$

$$= \frac{3}{8}$$

$$= 0.375$$

$$= 37.5\%$$

Probability for Event C

$$P(C) = \frac{\text{\# successes}}{\text{\# possibilities}}$$

$$= \frac{4}{8} = \frac{1}{2} \text{ (reduce to simplest terms)}$$

$$= 0.5$$

$$= 50\%$$

Remember $P(\text{event}) = \frac{\# \text{ successful outcomes}}{\# \text{ possible outcomes}}$ Date: _____

EXAMPLE 1

A standard deck of playing cards has 52 cards, 13 of each suit. If one card is drawn from the deck, find the probability of each event.

a) a heart $P(\text{heart}) = \frac{\# \text{ hearts}}{\# \text{ total cards}}$
 $= \frac{13}{52} = \frac{1}{4}$

b) an ace $P(\text{ace}) = \frac{\# \text{ aces}}{\# \text{ cards}} = \frac{4}{52} = \frac{1}{13}$

be careful not to count jacks twice; two are already included with the hearts and clubs

c) a heart, a club, or a jack

$$P(\text{heart, club, or jack}) = \frac{\# \text{ hearts} + \# \text{ clubs} + \# \text{ jacks}}{\# \text{ cards}} = \frac{13 + 13 + 2}{52} = \frac{28}{52} = \frac{7}{13}$$

d) a heart, a club, a spade, or a diamond

$$P(\text{heart, club, spade, diamond}) = \frac{\# \text{ hearts} + \# \text{ clubs} + \# \text{ spades} + \# \text{ diamonds}}{\# \text{ cards}}$$

$$= \frac{13 + 13 + 13 + 13}{52} = \frac{52}{52} = 1$$

EXAMPLE 2

Jason rolls a regular six-sided die. Find the theoretical probability of each event. Express your answer as a fraction in lowest terms and as a percent.

a) rolling a 6 $P(\text{roll a 6}) = \frac{\# \text{ sixes}}{\# \text{ sides}}$
 $= \frac{1}{6} = 16.7\%$

b) rolling a number greater than 2 $P(\text{roll} > 2) = \frac{\# \text{ sides} > 2}{\# \text{ sides}}$
 $= \frac{4}{6} = \frac{2}{3} = 66.7\%$

c) rolling an 8 $P(\text{roll 8}) = \frac{\# \text{ sides w/ 8}}{\# \text{ sides}}$
 $= \frac{0}{6} = 0\%$

d) rolling an even number

$$P(\text{roll even}) = \frac{\# \text{ sides that are even}}{\# \text{ sides}}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2} = 50\%$$