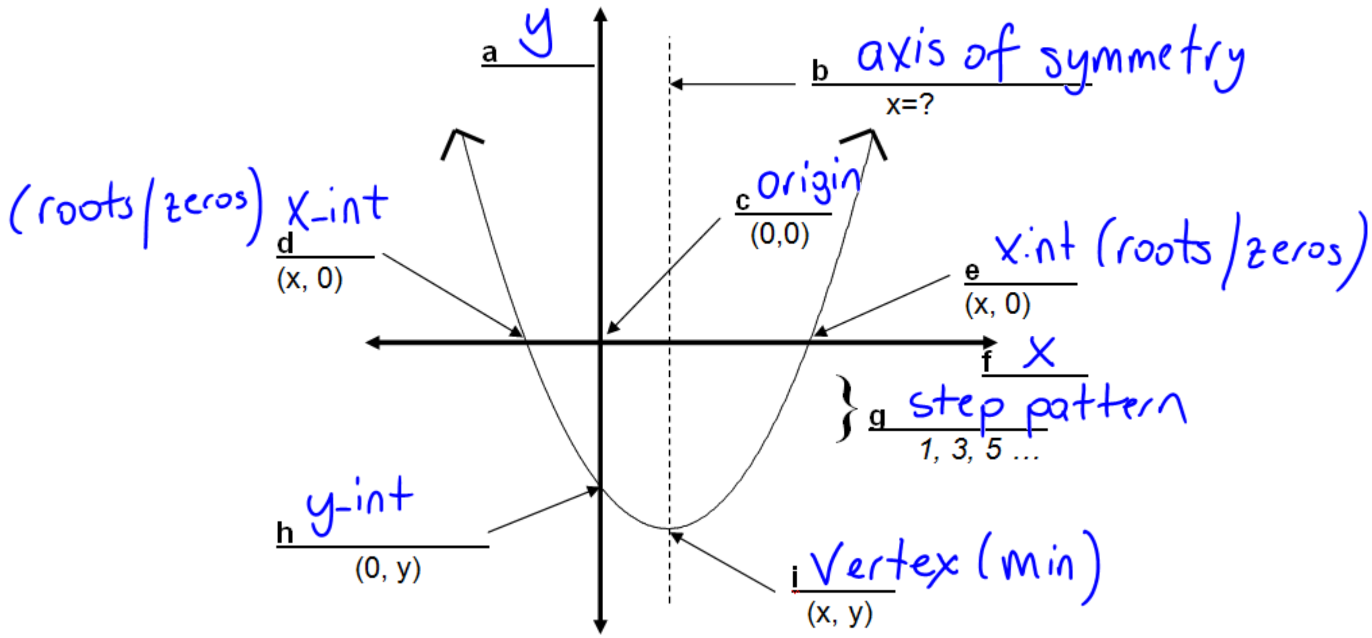
Day 1: Properties & Forms of the Quadratic Functions

QUADRATIC FUNCTION: $f(x) = ax^2 + bx + c$ **QUADRATIC EQUATION**: $0 = ax^2 + bx + c$



A. **Standard Form**:	B. **Vertex form**:	C. **Factored Form**:
$f(x) = ax^2 + bx + c$ • Shows direction of opening • Best form if you need the y-intercept (let x = 0)	$f(x) = a(x-h)^2 + k$ • Shows direction of opening Best form if you need the max/min value (k) and when it occurs (when $x = h$)	$y = a(x-r)(x-s)$ • Shows direction of opening • Best form if you need x-intercepts (let $y = 0$) • If you want to determine the max/min you must first expand it and write it in standard form and then complete the square

PROPERTIES OF QUADRATIC FUNCTIONS

Forms of Quadratic Functions

Standard form: $y = ax^2 + bx + c$ — y-intercept: (0, c)

Factored form: $y = a(x - p)(x - q)$ — roots, zeros, x-intercepts: (p, 0), (q, 0)

Vertex form: $y = a(x - h)^2 + k$ — vertex: (h, k)

Direction of opening – either up or down

If $a > 0$, the parabola opens up; if $a < 0$, the parabola opens down.

Roots – x values where the equation is equal to zero; the solutions to $ax^2 + bx + c = 0$

Zeros – x-intercepts; where the parabola crosses the x-axis; where the function is equal to zero

To determine roots/zeros, set the function equal to zero and solve by factoring or by using the quadratic formula. You can also solve by completing the square.

* There will be either 0, 1, or 2 roots/zeros!

Vertex – "turning point" of the parabola

- To determine the vertex from standard form, complete the square to write the equation in vertex form.
- To determine the vertex from factored form, find the x value that is halfway between the 2 zeros using the mean formula: $x = \frac{x_1 + x_2}{2}$. To find the y-coordinate, substitute the mean x value into the original equation [$y = a(x - p)(x - q)$ *or* $y = ax^2 + bx + c$]. The vertex is (x, y).

Optimal Value – the maximum or minimum value; the optimal value is the y value of the vertex

Axis of symmetry

– The parabola is *symmetric* with respect to this line

- If the coordinates of the vertex are (h,k), then the axis of symmetry is $x = h$.

To determine the axis of symmetry, find the mean x value for two points with equal y-values.

Methods for Writing Quadratic Functions in Different Forms

1. Each function is given in **standard** form. Factor fully to write the functions in **factored** form $f(x)=a(x-r)(x-s)$:

a) $f(x)=3x^2+6x$
b) $f(x)=x^2+x-20$
c) $f(x)=-2x^2-4x+6$
d) $f(x)=2x^2-9x+4$

a) $f(x)=3x^2+6x$ GCF $=3x$
$=3x(x+2)$

b) $f(x)=x^2+x-20$
$=(x-4)(x+5)$
M: $1\times-20=-20$
A: 1
N: $-4, +5$

c) $f(x)=-2x^2-4x+6$
$=-2(x^2+2x-3)$
$=-2(x-1)(x+3)$
M: $1\times-3=-3$
A: 2
N: $-1, +3$

d) $f(x)=2x^2-9x+4$
$=(2x-1)(x-4)$
M: $2\times4=8$
A: -9
N: $-1, -8$

$\frac{(2x-1)(2x-8)}{2}$
$\Downarrow$
$\frac{(2x-1)(\cancel{2})(x-4)}{\cancel{2}}$
$\Downarrow$
$(2x-1)(x-4)$

2. Each function is given in **standard** form. Complete the square to write the functions in **vertex** form $f(x)=a(x-h)^2+k$:

a) $f(x)=3x^2+6x$

b) $f(x)=x^2-8x+1$

c) $f(x)=-2x^2-4x+6$

d) $f(x)=2x^2-9x+4$

a) $f(x)=3x^2+6x$

$=3(x^2+2x)$ $\quad \frac{2}{2}=1 \quad 1\times1=1$

$=3(x^2+2x+1-1)$

$=3(x^2+2x+1)-3$

$=3(x+1)^2-3$

b) $f(x)=x^2-8x+1$ $\quad -\frac{8}{2}=-4 \quad -4\times-4=16$

$=(x^2-8x+16-16)+1$

$=(x^2-8x+16)-16+1$

$=(x-4)^2-15$

c) $f(x)=-2x^2-4x+6$

$=-2(x^2+2x)+6$ $\quad \frac{2}{2}=1 \quad 1\times1=1$

$=-2(x^2+2x+1-1)+6$

$=-2(x^2+2x+1)+2+6$

$=-2(x+1)^2+8$

d) $f(x)=2x^2-9x+4$

$=2(x^2-\frac{9}{2}x)+4$ $\quad \frac{9}{2}\div2=\frac{9}{2}\times\frac{1}{2}=\frac{9}{4} \quad \frac{9}{4}\times\frac{9}{4}=\frac{81}{16}$

$=2(x^2-\frac{9x}{2}+\frac{81}{16}-\frac{81}{16})+4$

$=2(x^2-\frac{9x}{2}+\frac{81}{16})-(\frac{81}{8}+4)$ $\quad \frac{81}{8}+\frac{4(8)}{1(8)}=\frac{81+32}{8}=\frac{113}{8}$

$=2(x-\frac{9}{4})^2-\frac{113}{8}$

3. Expand to re-write each function in **standard** form $f(x) = ax^2 + bx + c$:

a) $f(x) = (2x-1)(x+1)$
b) $f(x) = -3(x-1)(x+1)$
c) $f(x) = -(3x-2)(2x+1)$
d) $f(x) = (x-1)^2 - 8$
e) $f(x) = -2(x+3)^2 + 4$

a) $f(x) = (2x-1)(x+1)$
$= 2x^2 + 2x - x - 1$
$= 2x^2 + x - 1$

D.O.S

b) $f(x) = -3(x-1)(x+1)$
$= -3(x^2 - 1)$
$= -3x^2 + 3$

c) $f(x) = -(3x-2)(2x+1)$
$= -(6x^2 + 3x - 4x - 2)$
$= -(6x^2 - x - 2)$
$= -6x^2 + x + 2$

d) $f(x) = (x-1)^2 - 8$
$= (x-1)(x-1) - 8$
$= x^2 - 2x + 1 - 8$
$= x^2 - 2x - 7$

e) $f(x) = -2(x+3)^2 + 4$
$= -2(x^2 + 6x + 9) + 4$
$= -2x^2 - 12x - 18 + 4$
$= -2x^2 - 12x - 14$