7.1 Investigate Properties of Similar Triangles

Geometric shapes are often used in construction and design. Not only are certain shapes naturally pleasing to the eye, but they are also often useful for their structural properties.

Consider the shed shown. What geometric shapes can you recognize? Identify at least one pair of
- **congruent figures**
- **similar figures**

congruent figures
- identical shapes
- same size

similar figures
- identical shapes
- different sizes

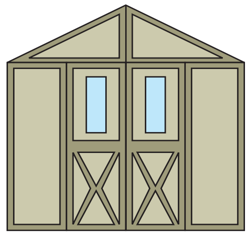
Tools
- copy of shed drawing
- ruler
- protractor

Optional
- tracing paper

Investigate

How can I recognize similar and congruent figures?

Look at the front face of the shed.



1. a) What pairs of congruent figures can you find?

b) Copy two or three pairs of congruent figures from the drawing of the shed. Label the vertices of each figure. What special properties do the corresponding sides and angles have?

2. Reflect Summarize the properties of congruent triangles.

3. a) What pairs of similar, but not congruent, figures can you find?

b) Make copies of two or three pairs of similar figures from the drawing of the shed. Label the vertices of each figure. What special properties do the corresponding sides and angles have?

4. Reflect

a) What is true about the corresponding angles in two congruent figures? in two similar figures?

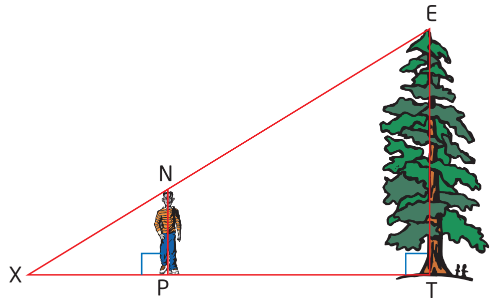
b) What is true about the corresponding side lengths in two congruent figures? in two similar figures?

c) Copy and complete the table to summarize your answers to parts a) and b).

	Corresponding Angles Are ...	Corresponding Side Lengths Are ...
Congruent Figures		
Similar Figures		

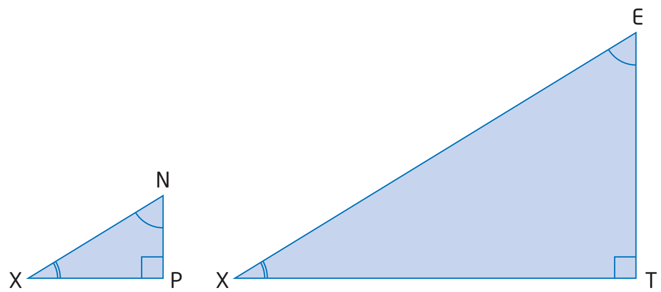
Example 1 Use Angles to Show That Two Triangles Are Similar

Identify a pair of similar triangles and explain why they are similar. Support each statement with a reason.



Solution

There are two overlapping similar triangles: △XPN and △XTE. The diagram shows them separated:



You can show that these two triangles are similar by identifying three pairs of equal angles.

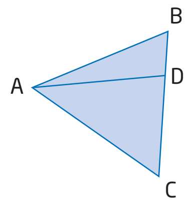
You can see three pairs of equal angles.

Statement	Reason
∠XPN = ∠XTE	Both are 90°.
∠PXN = ∠TXE	These are the same angle.
∠XNP = ∠XET	The sum of the angles in any triangle is 180°.
△XPN ~ △XTE	All three pairs of angles are equal.

The third pair of angles in any two triangles is equal if the other two pairs are equal. So, it is only necessary to have two pairs of equal corresponding angles to show that two triangles are similar.

Literacy Connections

In △ABC, the interior angle at vertex B can be called ∠ABC, ∠CBA, or ∠B. In the first two cases, the middle letter corresponds to the vertex of the angle.



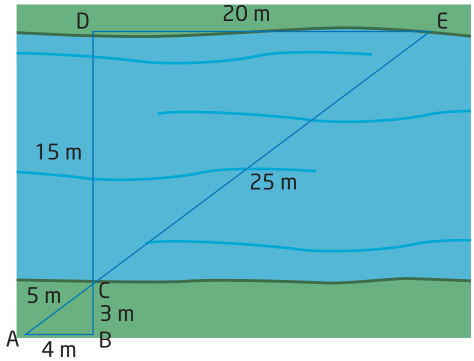
The three-letter system is useful to describe the three interior angles at vertex A: ∠BAD, ∠DAC, and ∠BAC. Referring simply to ∠A can lead to confusion.

Literacy Connections

The symbol ~ means "is similar to." When you write a similarity statement, the order of the vertices must correctly identify pairs of equal angles and pairs of corresponding sides. In Example 1, △XPN ~ △XTE is correct because the vertices correctly indicate pairs of equal angles and pairs of corresponding sides.

Example 2 Use Sides to Show That Two Triangles Are Similar

Are the two triangles in the diagram similar? Explain your reasoning.



Solution

You can show that two triangles are similar by showing that three pairs of corresponding sides are proportional.

The two triangles are △EDC and △ABC.

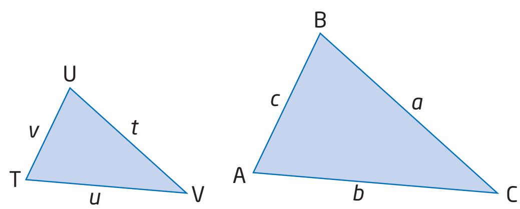
$$\frac{ED}{AB} = \frac{20}{4} = \frac{5}{1} \qquad \frac{CE}{CA} = \frac{25}{5} = \frac{5}{1} \qquad \frac{DC}{BC} = \frac{15}{3} = \frac{5}{1}$$

The three pairs of corresponding sides are all in the ratio 5:1. Therefore, △EDC ~ △ABC.

Since the triangles in Example 2 are similar, their corresponding angles are equal. That is, ∠CAB = ∠CED, ∠CBA = ∠CDE, and ∠ACB = ∠ECD.

Key Concepts

- Congruent figures have the same size and shape. Similar figures have the same shape, but different sizes.
- Similar triangles have the following properties:
  – Corresponding angles are equal:
  ∠A = ∠T
  ∠B = ∠U
  ∠C = ∠V
  – Ratios of corresponding sides are equal:



$$\frac{AB}{TU} = \frac{BC}{UV} = \frac{CA}{VT}$$, or, in ratio notation, AB:TU = BC:UV = CA:VT

Communicate Your Understanding

C1 **a)** Explain the difference between similar figures and congruent figures.

b) Draw an example for each.

C2 **a)** How are the angles of similar figures related?

b) How are the side lengths of similar figures related?

c) How do these relationships differ from those involving congruent figures?

C3 Describe two ways that you can show that two triangles are similar.

C4 Why is it only necessary to show that two pairs of corresponding angles are equal in order to show that two triangles are similar?

Practise

For help with questions 1 to 6, see the Investigate.

1. Look in the room around you. Identify at least two sets of

a) congruent figures

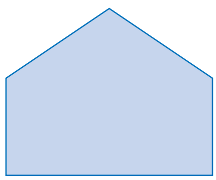
b) similar figures

2. Identify at least two examples outside the classroom of where you see

a) congruent figures

b) similar figures

3. **a)** Copy the diagram.



b) Draw each of the following:

- a figure congruent to the one in part a)
- a smaller figure that is similar to the one in part a)
- a larger figure that is similar to the one in part a)

c) Verify the accuracy of your sketches by measuring angles and side lengths.

4. For each situation, identify whether you would use similar figures, congruent figures, or neither. Justify your answers.

a) a tile pattern on a floor

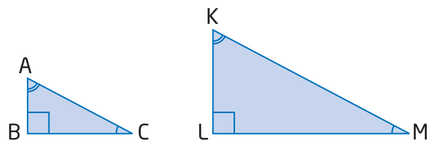
b) a team's logo for the chest and shoulder patches of their jerseys

c) a door frame and a window frame for the front of a house

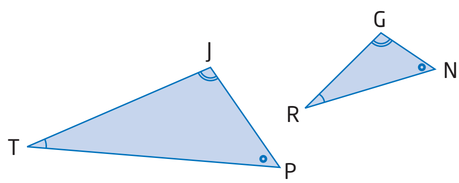
d) a three-dimensional model of a skyscraper building

5. Name the similar triangles in each case. Write the letters so that equal angles appear in corresponding order.

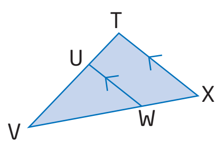
a)



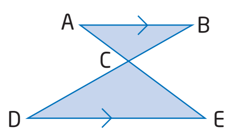
b)



c)



d)

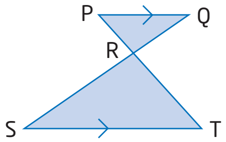


6. For each pair of similar triangles in question 5, write the equivalent ratios of side lengths.

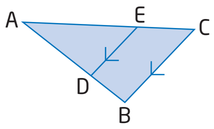
For help with questions 7 to 9, see Examples 1 and 2.

7. Name a pair of similar triangles in each diagram and explain why they are similar.

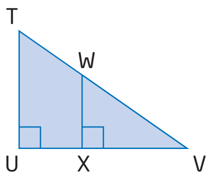
a)



b)

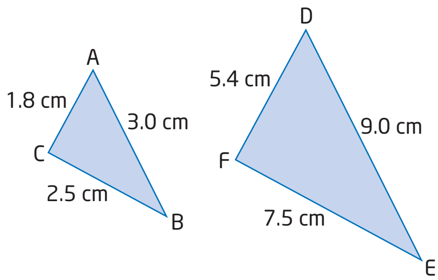


c)

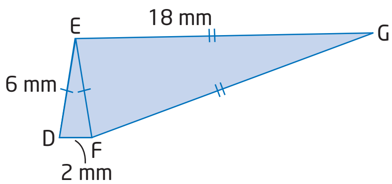


8. Name a pair of similar triangles in each diagram and explain why they are similar.

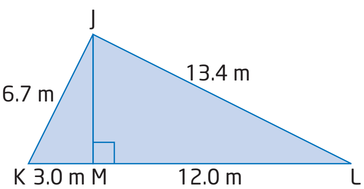
a)



b)



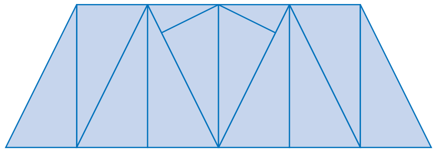
c)



9. For each pair of similar triangles in questions 7 and 8, list all the pairs of corresponding angles and corresponding sides.

Connect and Apply

Carefully copy or trace the diagram of the truss bridge. Use it to answer questions 10 to 12.



10. Identify an example of each of the following. Use different colours and/or additional sketches to illustrate your answers.

a) a pair of congruent triangles

b) a pair of congruent rectangles

c) a pair of congruent parallelograms

d) a pair of similar triangles

e) a set of three similar triangles

11. Identify a pair of congruent triangles that are related by a

a) translation

b) reflection

c) rotation

12. a) Describe the combination of transformations that relates one pair of congruent triangles.

b) Describe a different combination of transformations that relates another pair of congruent triangles.

13. a) Draw a right triangle.

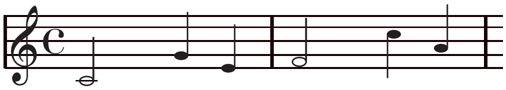
b) Draw a right triangle that is
- congruent to the one you drew
- similar to the one you drew
- neither congruent nor similar to the one you drew

14. Are all equilateral triangles similar? Justify your answer.

15. Are all isosceles triangles similar? Justify your answer.

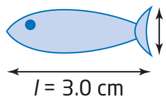
Extend

16. Congruent figures occur in music composition. This is a two-bar lead-in that was commonly used in the early days of rock 'n' roll music:



Carefully copy the music into your notebook. Then, use line segments to join the centres of the note heads (the round part) so they form two congruent triangles. What kind of transformation relates the two triangles?

17. A type of bass grows so that it retains its shape. Its length, height, and width all remain in the same proportion for its first few years of life. The average length, l, and height, h, of a baby bass are shown. The average width, w, of this type of bass is about half its height. If the bass triples in length each month for the first 3 months of life, find the dimensions of this bass after

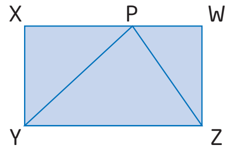


a) 1 month

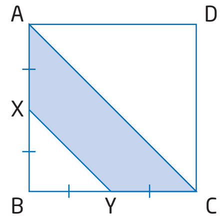
b) 2 months

18. Refer to question 17. The density of this type of bass is about 10% greater than that of water, which is 1 g/cm³. Use the relationship density $= \frac{\text{mass}}{\text{volume}}$ to estimate the mass of this bass after 3 months. To estimate the volume of the bass, find the volume of a rectangular prism with the same width, length, and height as the bass.

19. Math Contest XYZW is a rectangle, with YZ = 10 cm, YP = 8 cm, and ZP = 6 cm. Determine the area of XYZW.



20. Math Contest ABCD is a square. X is the midpoint of AB and Y is the midpoint of BC. What percent of the square is shaded?



A 37.5% **B** 25%

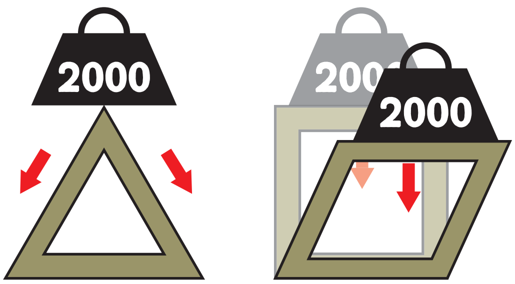
C 18.75% **D** $33\frac{1}{3}$% **E** 30.75%

Making Connections

The triangle is the strongest polygon structurally. The only way you can change its shape is to change the length of one or more of its sides. A square, by contrast, can be deformed into a rhombus under a heavy load.

It is this property that makes the triangle so popular in designs such as truss bridges.

The Quebec Bridge, shown here, which spans the St. Lawrence River, is the longest cantilever truss bridge in the world.



Go to www.mcgrawhill.ca/links/principles10 and follow the links to learn more about the Quebec Bridge and its fascinating history.