

# 7.2

## Use Similar Triangles to Solve Problems



The geometry of similar figures is a powerful area of mathematics. Similar triangles can be used to measure the heights of objects that are difficult to get to, such as trees, tall buildings, and cliffs. They can also be used to measure distances across rivers and even galaxies!

The students in the photo are using a metre stick and shadows to measure the height of the tree. How can they do this? What role do similar triangles play in this type of problem?

-  **Tools**
- ruler
  - metre stick

### Investigate

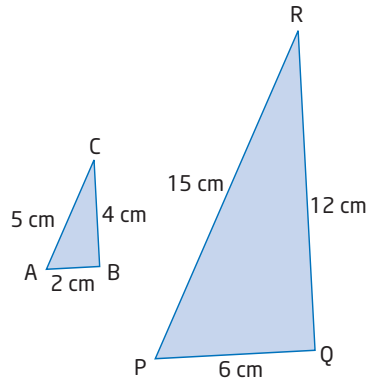
#### How can you apply the properties of similar triangles to solve problems?

Suppose you have a metre stick and it is sunny outside. Your task is to plan a problem solving strategy so that you can determine the height of an inaccessible object, such as a tree, a building, or a cliff.

1. Look at the illustration. Discuss with a partner, or in a small group, how the students could find the height of the tree.
2. Draw a diagram that relates to this problem. Explain how similar triangles are involved.
3. Create some numbers to represent reasonable measures to solve this problem and solve it. Does your answer seem reasonable? Explain.
4. **Reflect** Trade strategies with another pair or group. Compare strategies. Do you think they will work? Make any improvements you like to your own strategy. Later you will apply your method to solve a real measurement problem.

The **scale factor,  $k$** , is a useful quantity when working with similar triangles such as the ones shown.

The value of  $k$  relating corresponding sides in these two triangles is 3, because if you multiply each side length in  $\triangle ABC$  by 3, you obtain the corresponding side length in  $\triangle PQR$ .



**scale factor,  $k$**

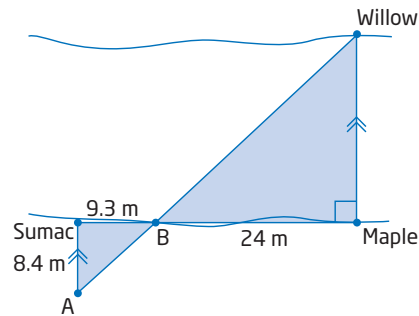
- factor that relates corresponding side lengths of two similar triangles

$\triangle ABC$ Side Lengths (cm)	Multiply by $k$	$\triangle PQR$ Side Lengths (cm)
AB = 2	$2 \times 3 = 6$	PQ = 6
BC = 4	$4 \times 3 = 12$	QR = 12
CA = 5	$5 \times 3 = 15$	RP = 15

You can apply the scale factor to find an unknown side length in one triangle if you know the corresponding side length in a similar triangle.

**Example 1 Solve for an Unknown Side**

To determine the width of a river, Naomi finds a willow tree and a maple tree that are directly across from each other on opposite shores. Using a third tree on the shoreline, Naomi plants two stakes, A and B, and measures the distances shown.



Find the width of the river using the information that Naomi found.

**Solution**

You can find two similar triangles and then use the scale factor to find the width.

*Step 1: Show that  $\triangle ABS$  is similar to  $\triangle WBM$ .*

**Statement**

$\angle ABS = \angle WBM$

$\angle BSA = \angle BMW$

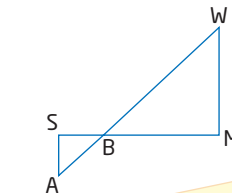
$\triangle ABS \sim \triangle WBM$

**Reason**

These are opposite angles.

These are both  $90^\circ$ , because AS is parallel to WM.

Two pairs of corresponding angles are equal.

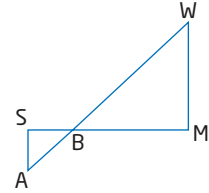


If two pairs of corresponding angles are equal, I know that the angles in the third pair are also equal.

*Step 2: Find the scale factor.*

In  $\triangle ABS$  and  $\triangle WBM$ ,  $BS$  and  $BM$  are corresponding sides. Their ratio gives the scale factor,  $k$ :

$$\begin{aligned} k &= \frac{BM}{BS} \\ &= \frac{24}{9.3} \end{aligned}$$



Each side in the larger triangle is  $\frac{24}{9.3}$  times as long as its corresponding side in the smaller triangle. Leave the scale factor in this form for more accuracy in later calculations.

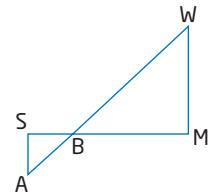
*Step 3: Find the width of the river.*

In  $\triangle ABS$  and  $\triangle WBM$ ,  $AS$  and  $WM$  are corresponding sides. Use the scale factor to find  $WM$ .

$$\frac{WM}{AS} = k$$

$$\frac{WM}{8.4} = \frac{24}{9.3}$$

$$\begin{aligned} WM &= \frac{24}{9.3} (8.4) && \text{Multiply both sides by 8.4.} \\ &\doteq 21.667 \end{aligned}$$

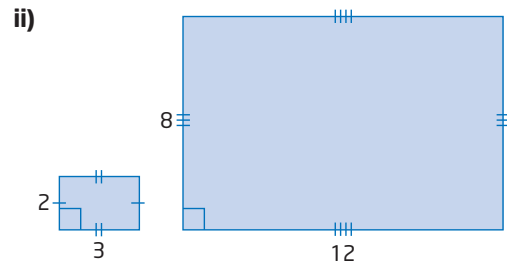
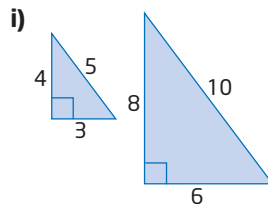


21.677 m is too precise an answer based on the measurements given. One of the measures,  $BM$ , is only accurate to the nearest metre. So, it is reasonable to round the answer to the nearest metre.

Therefore, the width of the river is approximately 22 m.

## Example 2 Areas of Similar Figures

a) What is the relationship between the areas in each pair of similar figures?



b) Find the scale factor,  $k$ , for each pair of figures.

c) Compare your answers to parts a) and b).

## Solution

a) Find the areas of the figures.

i) **Smaller Triangle**

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(3)(4) \\ &= 6 \end{aligned}$$

The area of the smaller triangle is 6 square units.

The area of the larger triangle is 4 times the area of the smaller triangle.

**Larger Triangle**

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(6)(8) \\ &= 24 \end{aligned}$$

The area of the larger triangle is 24 square units.

ii) **Smaller Rectangle**

$$\begin{aligned} A &= l \times w \\ &= 2 \times 3 \\ &= 6 \end{aligned}$$

The area of the smaller rectangle is 6 square units.

The area of the larger rectangle is 16 times the area of the smaller rectangle.

**Larger Rectangle**

$$\begin{aligned} A &= l \times w \\ &= 8 \times 12 \\ &= 96 \end{aligned}$$

The area of the larger rectangle is 96 square units.

b) i) Since each side length of the larger triangle is 2 times the length of the corresponding side of the smaller triangle, the scale factor is  $k = 2$ .

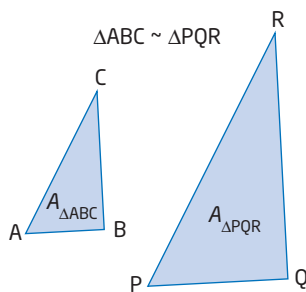
ii) Since each side length of the larger rectangle is 4 times the length of the corresponding side of the smaller rectangle, the scale factor is  $k = 4$ .

c) In both cases, the ratio of the area of the larger figure to the area of the smaller figure is equal to the square of the scale factor,  $k$ .

This relationship holds for all similar figures: the ratio of the areas of two similar figures is equal to the square of the scale factor.

Another way to write this is

$A_{\Delta PQR} = k^2(A_{\Delta ABC})$ . You can use this to solve for an unknown area.

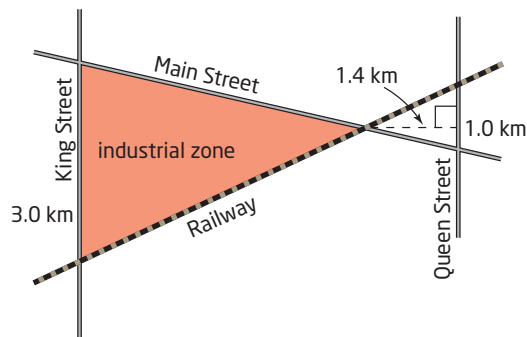


$$\frac{A_{\Delta PQR}}{A_{\Delta ABC}} = k^2$$

### Example 3 Solve for an Unknown Area

The shaded area is to be an industrial zone.

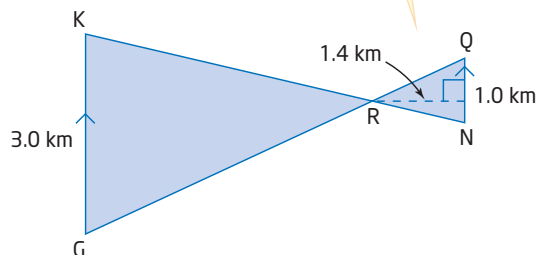
Find the area of the industrial zone. Assume that King and Queen are parallel and that all streets and the track are straight.



#### Solution

Identify two similar triangles.  
Find the scale factor and use it to find the area of the larger triangle.

I'll make a simplified diagram.



#### Statement

$\angle KRG = \angle NRQ$   
 $\angle RKG = \angle RNQ$   
 $\triangle KRG \sim \triangle NRQ$

#### Reason

Opposite angles are equal.  
 Alternate angles are equal.  
 Corresponding angles are equal.

If two pairs of corresponding angles are equal, I know that the angles in the third pair are equal, too.

The scale factor is equal to the ratio of corresponding sides:

$$\begin{aligned} k &= \frac{KG}{NQ} \\ &= \frac{3.0}{1.0} \\ &= 3 \end{aligned}$$

Find the area of the smaller triangle using the given information.

$$\begin{aligned} A_{\triangle NRQ} &= \frac{1}{2}bh && \text{Apply the formula for the area of a triangle.} \\ &= \frac{1}{2}(1.0)(1.4) \\ &= 0.7 \end{aligned}$$

The area of the smaller triangle is  $0.7 \text{ km}^2$ . Use this and the scale factor,  $k = 3$ , to find the area of the larger similar triangle.

$$\begin{aligned} A_{\triangle KRG} &= k^2(A_{\triangle NRQ}) \\ &= 3^2(0.7) \\ &= 9(0.7) \\ &= 6.3 \end{aligned}$$

The area of the industrial zone is  $6.3 \text{ km}^2$ .

## Key Concepts

- The scale factor,  $k$ , relates the lengths of corresponding sides of similar figures. For example, in  $\triangle ABC$  and  $\triangle PQR$ ,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = k$$

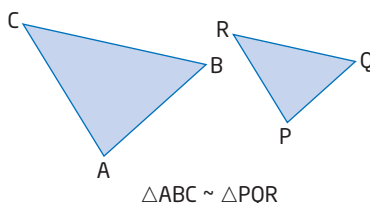
These relationships can also be written as follows:

$$AB = k(PQ) \quad BC = k(QR) \quad CA = k(RP)$$

- The square of the scale factor relates the areas of two similar figures:

$$\frac{A_{\triangle ABC}}{A_{\triangle PQR}} = k^2$$

This relationship can also be written as  $A_{\triangle ABC} = k^2(A_{\triangle PQR})$ .



## Communicate Your Understanding

- C1** a) Explain how the scale factor relates two similar triangles.  
b) Explain how you can use the scale factor to find an unknown side length.
- C2** a) How are the areas of two similar figures related?  
b) Explain using words and diagrams how you can find the area of a triangle using the area of a similar triangle and the scale factor.
- C3** Explain how you can use a metre stick and shadows to measure the height of an inaccessible object, such as a flagpole or a tree.

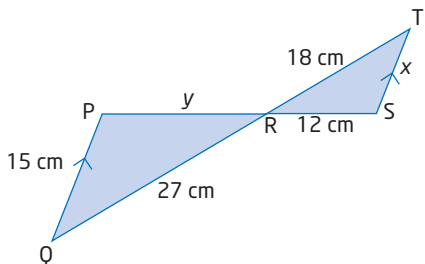
## Practise

- A right triangle has side lengths 3 cm, 4 cm, and 5 cm.
  - Draw the triangle.
  - A similar triangle has hypotenuse 30 cm long. What is the scale factor?
  - What are the lengths of the legs?
- Refer to question 1.
  - Find the area of each triangle.
  - How are these areas related?
  - How do the areas help to confirm that the triangles are similar?
- Draw a triangle.
  - Draw a similar triangle using a scale factor of 2.
  - Repeat part b) using a scale factor of 4.
- Refer to question 3.
  - Measure, as accurately as possible, the base and height of the first triangle. Use this information to find the area of the triangle. Round your answer to the nearest tenth.
  - Use your answer from part a) and the scale factors to calculate the areas of the two larger triangles.
  - Measure the base and height of each larger triangle and use them to calculate their areas. Compare these results with those obtained in part b). Are they the same? If they are not the same, describe what factors might explain why not.

For help with questions 5 to 7, see Example 1.

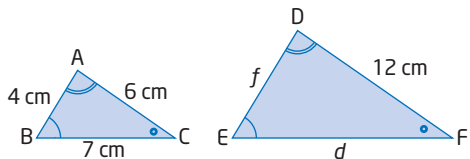
5. a) Show why  $\triangle PQR$  is similar to  $\triangle STR$ .

b) Find the lengths  $x$  and  $y$ .

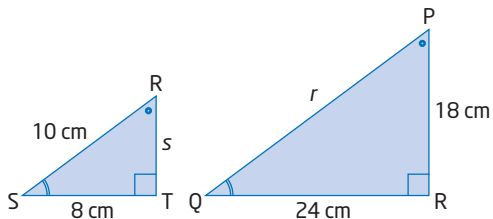


6. The triangles in each pair are similar. Find the unknown side lengths.

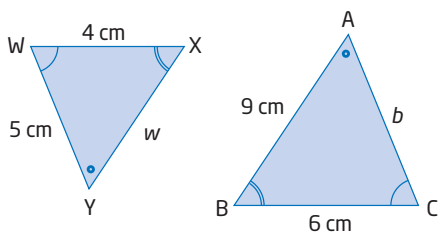
a)



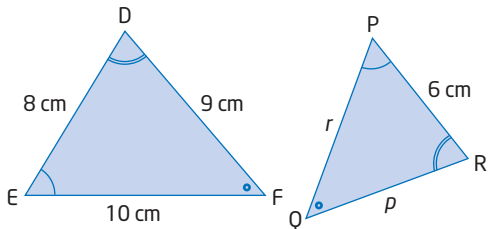
b)



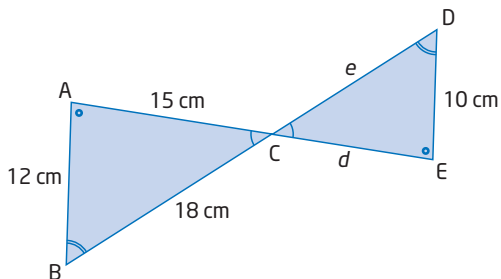
c)



d)

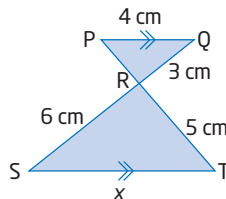


e)

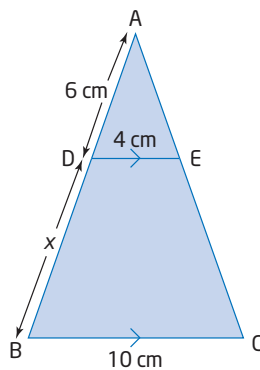


7. Find the length of  $x$  in each.

a)

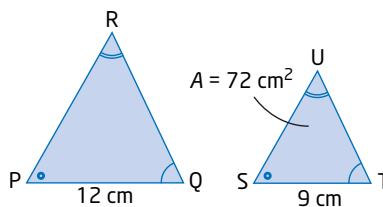


b)

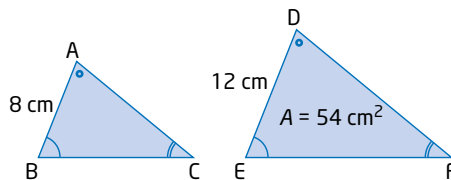


For help with question 8, see Examples 2 and 3.

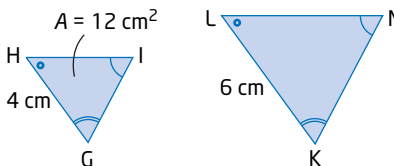
8. a)  $\triangle PQR \sim \triangle STU$ . Find the area of  $\triangle PQR$ .



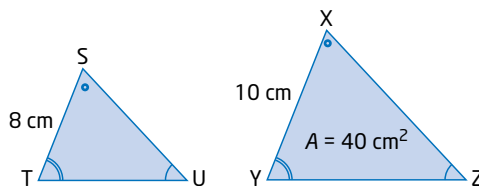
b)  $\triangle ABC \sim \triangle DEF$ . Find the area of  $\triangle ABC$ .



c)  $\triangle GHI \sim \triangle KLM$ . Find the area of  $\triangle KLM$ .

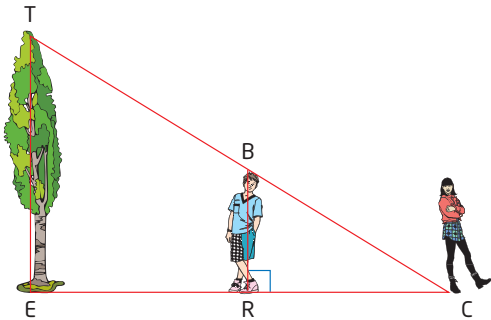


d)  $\triangle STU \sim \triangle XYZ$ . Find the area of  $\triangle STU$ .



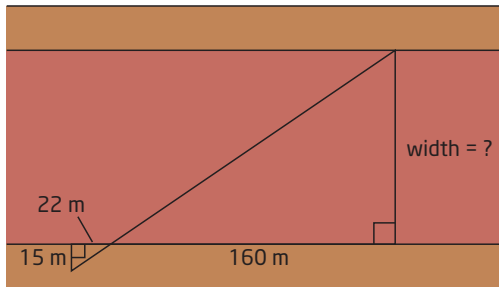
## Connect and Apply

9. To measure the height of a tree, Cynthia has her little brother, BR, stand so that the tip of his shadow coincides with the tip of the tree's shadow, at point C.

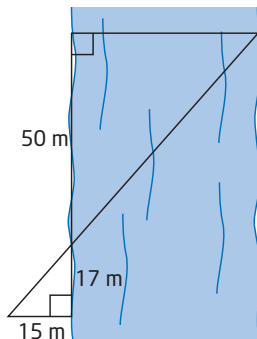


Cynthia's brother, who is 1.2 m tall, is 4.2 m from Cynthia, who is standing at C, and 6.5 m from the base of the tree. Find the height of the tree, TE.

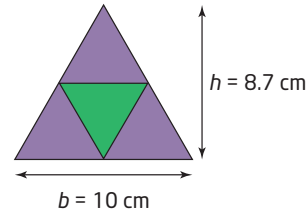
10. Find the width of the canyon.



11. Use the dimensions of the surveyors' triangles to find the width of the river, to the nearest metre.

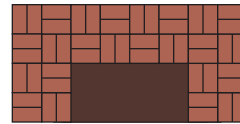


12. Melanie is designing a crest for her hockey team, the Trigazoids. Her prototype consists of four congruent equilateral triangles.



- What is the total area of this crest?
- What is the area of
  - the green section?
  - the purple sections?
- What is the area of a giant similar crest with base 30 cm?
- What is the height of a similar crest with area  $500 \text{ cm}^2$ ?

13. The front of each brick in the fireplace measures 10 cm by 20 cm.



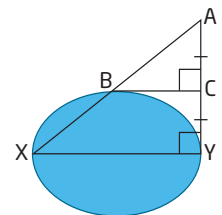
- How many similar rectangles of different sizes can you find? Sketch a diagram to illustrate them. Label their dimensions (length and width).
- What is the area of the front of one brick?
- Find the area of the entire fireplace, including the opening.
- Find the area of the opening.
- Find the area of the fireplace, excluding the opening.

14. Find the length and width of the pond.
- The following measures are known:

$$AB = 14 \text{ m}$$

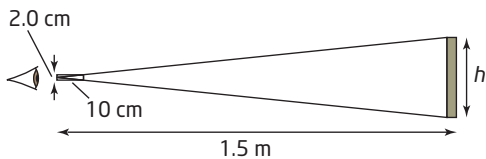
$$BC = 11 \text{ m}$$

Assume that XY is a line of symmetry for the pond.





15. Determine the height of a tall tree, a flagpole, or the side of a building in your schoolyard using similar triangles. Explain your method using words and diagrams.
16. While looking through a cylindrical tube, Rita moves to a point where the height of a picture just fits within her field of view, as shown.



Rita is standing 1.5 m from the picture. The length and diameter of the viewing tube are as shown. Find the height of the picture.

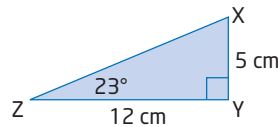
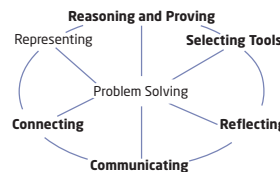
17. Use algebraic and geometric reasoning to show how the areas of two similar right triangles are related by the square of the scale factor,  $k^2$ .
18. a) Sketch several pairs of similar acute triangles with different scale factors,  $k$ .  
 b) Find the areas of the triangles in each pair.  
 c) Find the ratio of the areas of the triangles in each pair. How is this ratio related to the scale factor,  $k$ ?
19. The areas of two similar triangles are  $72 \text{ cm}^2$  and  $162 \text{ cm}^2$ . What is the ratio of the lengths of their corresponding sides?
20.  $\triangle ABC$  and  $\triangle DEF$  are similar. The ratio of their corresponding sides is 3:5. What is the ratio of their perimeters? Explain.
21. Use similar triangles to measure the height of the building in which you live. Write a brief report on how you solved this problem. Include diagrams. Discuss how accurate you think your answer is. Suggest ways to improve your method to get a more accurate height.

22. **Chapter Problem** The first leg of your race will begin on the southern shore of James Bay, at Moosonee. From there you will travel to Regina, then to Churchill, located on the eastern shore of Hudson Bay. Take note of your journey. The triangle formed by these three locations is similar to the triangle formed by Pittsburgh, Repulse Bay (located near the Arctic Circle), and your next destination. Identify the similar triangles and determine your next destination. *Hint: Move quickly, and you will be glad that you beat the rest of the flock!*



**Achievement Check**

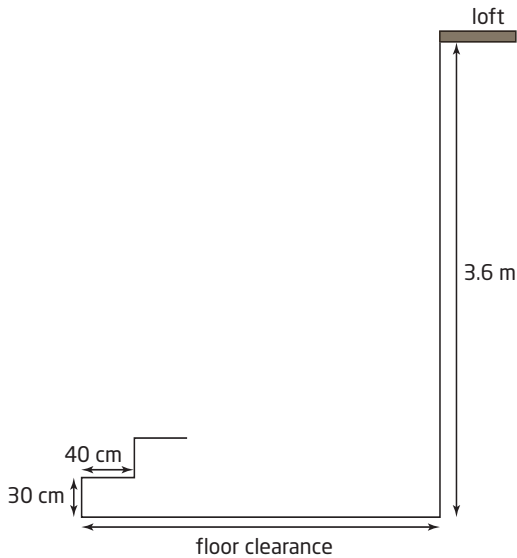
23. Teschia is making a scale drawing to help her redesign her flower garden.



- a) Calculate the length of  $ZX$  and the measure of  $\angle ZXY$ .
- b) If the hypotenuse of the actual flower garden measures 6.5 m, what is the perimeter of the actual garden?
- c) What is the scale factor of Teschia's drawing?
- d) What is the ratio of the area of the flower garden to the area of the scale drawing?

## Extend

24. Carol is building a staircase from the floor of her barn to the loft, which is 3.6 m above the floor. She is using steps that are each 30 cm high and 40 cm deep.

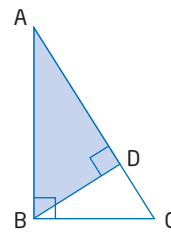


- a) How much floor clearance will Carol need in order to fit the staircase?  
 b) How many steps will be required?
25. The scale on a map is 1 cm represents 5 km. A provincial park has an area of  $6 \text{ cm}^2$  on the map. What is the actual area of the park, to the nearest square kilometre?
26. Krista used her Global Positioning System (GPS) device to obtain information on the distance and direction from Niagara Falls to London, England, and to Miami, Florida. She drew a triangle, and calculated the angles in the triangle from the GPS data. She noticed that the sum of the angles was not  $180^\circ$ , as expected.
- a) Why did this occur?  
 b) Would you expect the sum to be more or less than  $180^\circ$ ? Explain.

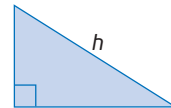
27. **Math Contest** A naturalist's study in Northern Ontario finds that 25% of the area is water and 60% of the remaining area is forest. The rest, 12 000 ha, is rock. How large is the study area, in hectares?

- A 36 000 ha  
 B 40 000 ha  
 C 68 000 ha  
 D 80 000 ha  
 E 100 000 ha

28. **Math Contest** In  $\triangle ABC$ ,  $AB = 24 \text{ cm}$  and  $BC = 10 \text{ cm}$ .  $BD$  is perpendicular to  $AC$ . Find the ratio of the shaded area to the unshaded area.



29. **Math Contest** Express the length of the hypotenuse of a right triangle in terms of its area,  $A$ , and its perimeter,  $P$ .



30. **Math Contest** Two neighbouring houses are located at  $A$  and  $B$ , near a straight section of a rural road,  $RD$ . The electric company plans to place a pole,  $P$ , at the roadside and connect wires from the pole to the two houses. How far from point  $R$  should the pole be located so that the minimum length of wire is needed?

