

Investigation: Exponent Rules

Complete the following table, using what you know about exponents and the example provided.

RULE 1 – MULTIPLICATION OF POWERS WITH THE SAME BASE

Question	Repeated Multiplication	Answer in Exponential Form	Exponent of Answer	Exponents in Original Question
$2^3 \times 2^5$		2^8	8	3, 5
$5^5 \times 5^5$	$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$	5^{10}	10	5, 5
$x^3 \times x^2$	$x \cdot x \cdot x \cdot x \cdot x$	x^5	5	3, 2
$\left(\frac{3}{4}\right)^2 \times \left(\frac{3}{4}\right)^5$	$\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)$	$\left(\frac{3}{4}\right)^7$	7	2, 5

Compare the exponents in the answer and in the original question.
To multiply powers, you ADD the exponents, if the base is the same.

RULE 2 – DIVISION OF POWERS WITH THE SAME BASE

Question	Repeated Multiplication	Answer in Exponential Form	Exponent of Answer	Exponents in Original Question
$\frac{10^5}{10^3}$	$\frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10}$ $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$	10^2	2	5, 3
$\frac{7^8}{7^5}$	$7 \times 7 \times 7 \times 7 \times 7$ $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$	7^3	3	8, 5
$\frac{x^6}{x^5}$	$x \cdot x \cdot x \cdot x \cdot x \cdot x$ $x \cdot x \cdot x \cdot x \cdot x$	x^1	1	6, 5

Compare the exponents in the answer and in the original question.
To divide powers, you SUBTRACT MINUS TAKE-AWAY the exponents, if the base is the same.

$$\frac{x^6}{x^5} = x^{6-5} = x^1$$

Simplify using the exponent rules.

Example 1:

$$2^2 \times 2^5$$

$$= 2^{2+5}$$

$$= 2^7$$

Example 2:

$$y^{11} \div y^7$$

$$= y^{11-7}$$

$$= y^4$$

Example 3:

$$\frac{(-2)^3 (8)^5 (-2)^8 (8)^1}{(-2)^4 (8)^3}$$

like exponents

$$= \frac{(-2)^{3+8} (8)^{5+1}}{(-2)^4 (8)^3}$$

$$= \frac{(-2)^{11} (8)^6}{(-2)^4 (8)^3}$$

$$= (-2)^{11-4} \cdot (8)^{6-3}$$

$$= (-2)^7 \cdot (8)^3$$

$$= -65536$$

Example 4:

$$\frac{(-2)^8 \cdot (-2)^{-3} \cdot (-3)^2}{(-2)^2 \cdot (-3)^{-1}} = \frac{(-2)^{8-3} \cdot (-3)^{2-(-1)}}{(-2)^{2-2} \cdot (-3)^{-1-(-1)}}$$

$$= \frac{(-2)^5 \cdot (-3)^3}{(-2)^0 \cdot (-3)^0}$$

$$= (-2)^5 \cdot (-3)^3$$

RULE 3 – POWER OF A POWER

Complete the following table, using what you know about exponents, the multiplication rule, and the example provided.

Question	Repeated Multiplication	Answer in Exponential Form	Exponent of Answer	Exponents in Original Question
$(2^3)^4$	$2^3 \times 2^3 \times 2^3 \times 2^3$	2^{12} (from multiplication rule)	12	3, 4
$(3^6)^2$	$3^6 \times 3^6$	$3^{6+6} = 3^{12}$	12	6, 2
$(x^5)^3$	$x^5 \cdot x^5 \cdot x^5$	$x^{5+5+5} = x^{15}$	15	5, 3

Compare the exponents in the answer and in the original question.
To raise a power to a power, you MULTIPLY the exponents, if the base is the same.

KEEP THE BASE, MULTIPLY THE POWERS

Example 1: $(x^4)^5$
 $= x^{4 \times 5}$
 $= x^{20}$

Example 2: $(2^3)^2$
 $= 2^{3 \times 2}$
 $= 2^6 = \underline{\underline{64}}$

Example 3: $(x^5y)^2$

METHOD 1 } $= (x^5y)(x^5y)$
 $= x^{5+5} \cdot y^{1+1}$
 $= x^{10} \cdot y^2$

(OR)

Example 4: $(xy^2)^2$

$= (xy^2)(xy^2)$
 $= \underline{\underline{x^2y^4}}$

METHOD 2 } Multiply each exponent by 2

$= x^{5 \times 2} \cdot y^{1 \times 2}$
 $= x^{10} \cdot y^2$

Practice: Exponent Rules

Simplify, but do not evaluate

<p>a. $8^3 \times 8^6$ $= 8^{3+6}$ $= 8^9$</p>	<p>b. $y^3 \times y^4 \times y$ $= y^{3+4+1}$ $= y^8$</p>	<p>c. $(-6)^2 \times (-6)^4$ $= (-6)^{2+4}$ $= (-6)^6$</p>	<p>d. $2^3 \times 4^2 \times 4 \times 2^5$ $= 2^{3+5} \times 4^{2+1}$ $= 2^8 \times (4^3)$ $= 2^8 \times (2^2)^3$ $= 2^8 \times 2^6 = 2^{14}$</p>
<p>e. $5^3 \div 5^2 \times 5^8$ $= 5^{3-2} \times 5^8$ $= 5^1 \times 5^8$ $= 5^{1+8}$ $= 5^9$</p>	<p>f. $8^4 \times 8^3 \div 8^5$ $= 8^{4+3} \div 8^5$ $= 8^7 \div 8^5$ $= 8^{7-5}$ $= 8^2$</p>	<p>g. $\left(\frac{3}{2}\right)^2 \times \left(\frac{3}{2}\right)^5$ $= \left(\frac{3}{2}\right)^{2+5}$ $= \left(\frac{3}{2}\right)^7$</p>	<p>h. $\frac{2^2 \times 3^2 \times 2^4 \times 3}{2^5 \times 3}$ $= \frac{2^{2+4} \times 3^{2+1}}{2^5 \times 3}$ $= \frac{2^6 \times 3^3}{2^5 \times 3}$ $= 2^{6-5} \times 3^{3-1} = 2 \times 3^2$</p>
<p>i. $(5^2)^3$ $= (5^2)(5^2)(5^2)$ $= 5^{2+2+2}$ $= 5^6$</p>	<p>j. $(a^3b)^2$ $= a^{3 \times 2} \cdot b^{1 \times 2}$ $= a^6 b^2$</p>	<p>k. $\frac{a^3b^6}{ab^2} = a^{3-1} \cdot b^{6-2}$ $= a^2 \cdot b^4$</p>	<p>l. $(m^2n)^2 = m^{2 \times 2} \cdot n^{1 \times 2}$ $= m^4 n^2$</p>

Find the missing exponent:

<p>m. $10^6 \times 10^x = 10^{10}$ if $10^{6+x} = 10^{10}$ then $6+x = 10$ $x = 10-6$ $x = 4$</p>	<p>n. $\frac{5^x}{5^3} = 5^2$ $5^{x-3} = 5^2$ $x-3 = 2$ $x = 2+3$ $x = 5$</p>	<p>o. $3^x \times 3^3 = 3^7$ if $3^{x+3} = 3^7$ then $x+3 = 7$ $x = 7-3$ $x = 4$</p>	<p>p. $\frac{(-2)^8}{(-2)^x} = (-2)$ if $(-2)^{8-x} = (-2)^1$ then $8-x = 1$ $-x = 1-8$ $-x = -7$ $x = 7$</p>
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ANSWERS: a) 8^9 , b) y^8 , c) $(-6)^6$, d) $2^8 \times 4^3$, e) 5^9 , f) 8^2 , g) $(3/2)^7$, h) 2×3^2 , i) 5^6 , j) $a^6 b^2$, k) $a^2 b^4$, l) $m^4 n^2$, m) $x = 4$, n) $x = 5$, o) $x = 4$, p) $x = 7$