

The FIBONACCI SEQUENCE and RECURSION

Who is Fibonacci?

He was an Italian mathematician, considered to be "the most talented Western mathematician of the Middle Ages". His real name was Leonardo Pisano Bogollo, and lived between 1170 and 1250 in Italy. "Fibonacci" was his nickname, which roughly means "Son of Bonacci".

As well as being famous for the Fibonacci Sequence, he helped spread Hindu - Arabic Numerals (our present numbers 0, 1, 2, 3, 4, etc.) through Europe in place of Roman Numerals (I, II, III, etc.)



What is Fibonacci Sequence?

0, 1, 1, 3, 5, 8, 13, 21, 34 ...

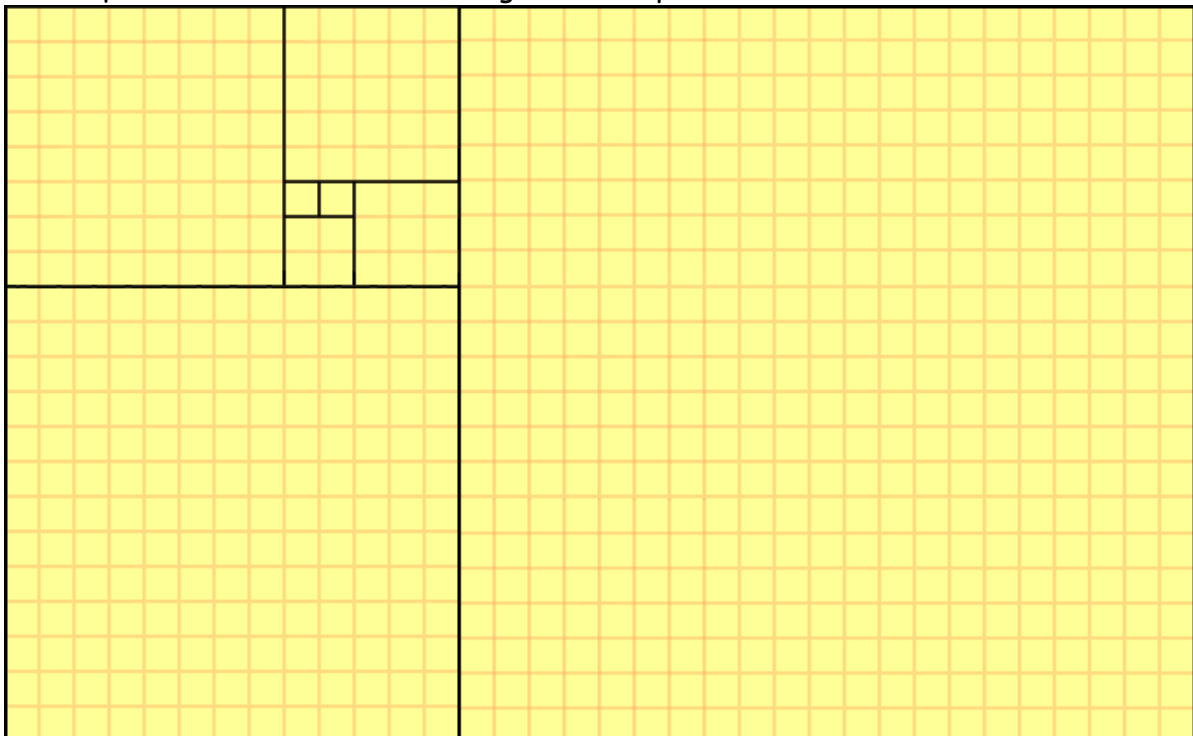
The next number is found by adding up the two numbers before it. Therefore, the next number in the sequence above is _____

Fibonacci Day

It is November 23rd, as it has the digits "1, 1, 2, 3" which is part of the sequence.

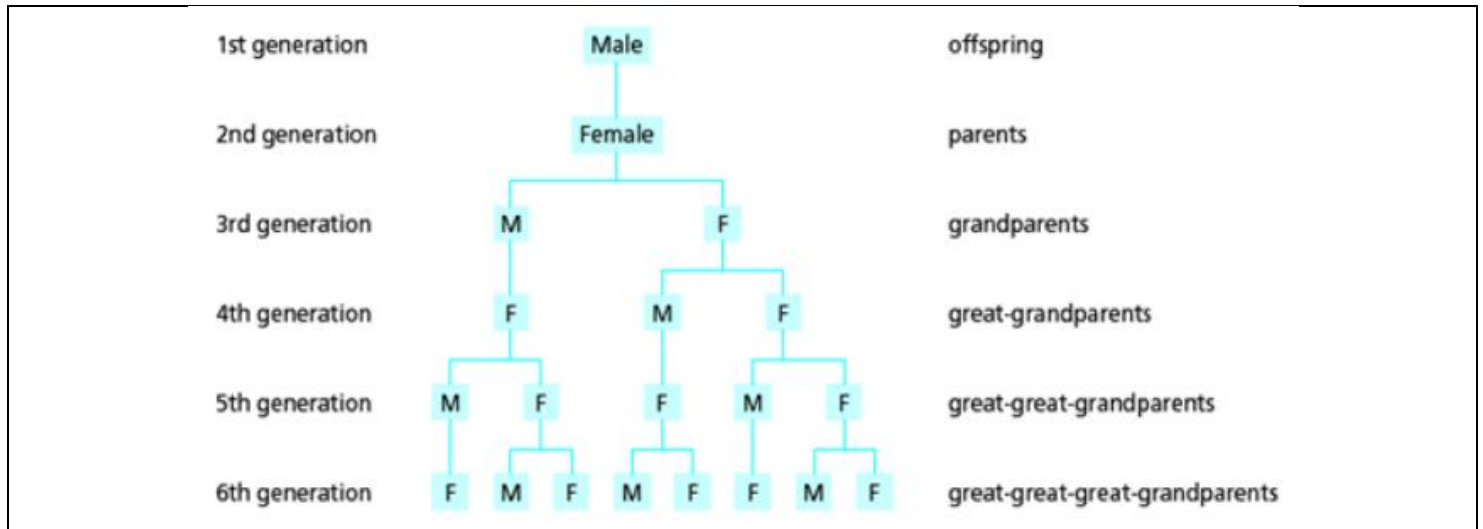
Makes a Spiral

When we make squares with those widths, we get a nice spiral:



A Real Life Example: Honeybees

There are many examples of sequences in nature, and in our everyday lives. A female honeybee hatches from an egg laid by a female honeybee, after the egg has been fertilized by the male. So each female honeybee has two parents. However, a male honeybee hatches from an unfertilized egg and has only one parent, a female honeybee. The tree diagram shows six generations of a typical male honeybee.



- Starting with the first generation, write the sequence of the number of honeybees in each generation.
- Identify the pattern in this sequence.
- Use this pattern to determine the next five terms of the sequence.
- Let t_n represent any term in this sequence. How could you represent the term before t_n ? How could you represent the term before the term that is before t_n ?
- Given that $t_1 = 1$, $t_2 = 1$, and n is a natural number, determine a formula for the general term of this sequence.

Key Ideas

- In a **recursive sequence**, a new term is generated from the previous term or terms. For example, 1, 2, -1, 3, -4, 7, -11, ... is a recursive sequence because each term beginning with the third term, is the result of subtracting t_{n-1} from t_{n-2} .
- A **recursive formula** shows how to find each term from the previous term or terms. For example,
 $t_1 = 1, t_2 = 2, t_n = t_{n-2} - t_{n-1}$, where n is a natural number.
To find t_n , subtract t_{n-1} from t_{n-2} . The first two terms are given.
The sequence is 1, 2, -1, 3, -4, 7, -11,
- A recursive formula refers to at least one known term. The first term, and sometimes several other terms, appear with the formula. Examine the formula carefully before applying it.

Ex1. Write the first four terms of each sequence.

a) $t_1 = 2, t_n = 3t_{n-1} + 5$

b) $t_1 = -1, t_2 = 1, t_n = 2t_{n-2} + 4t_{n-1}$

Ex2. Write a recursive formula for each sequence.

a) 1, 2, 4, 7, 11, 16

b) 4, 5, 20, 100, 2000, ...

c) -400, 200, -100, 50, ...