

## Discovering the Slope Formula

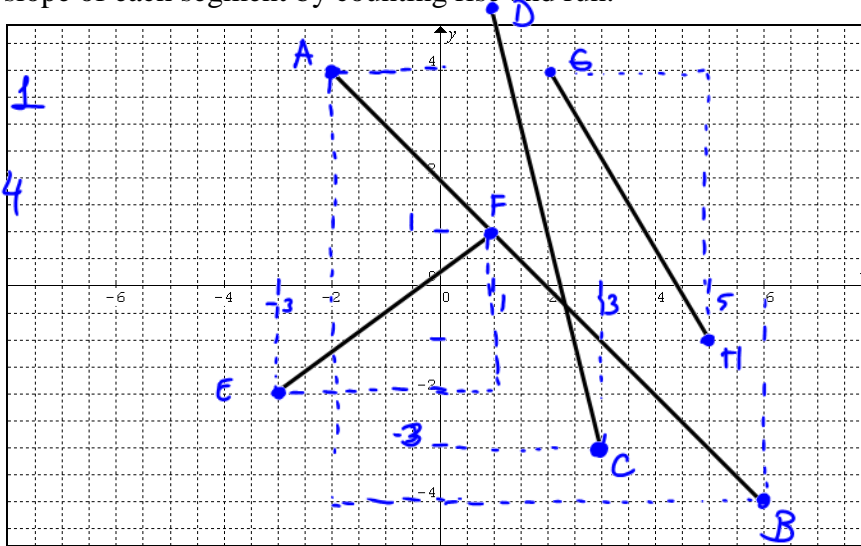
- Plot the following points and draw the segment created by joining the points.
  - A(-2, 4) B(6, -4)
  - C(3, -3) D(1, 5)
  - E(-3, -2) F(1, 1)
  - G(2, 4) H(5, -1)
- Determine the slope of each segment by counting rise and run.

$$\text{slope } \overline{AB} = \frac{-8}{8} = -1$$

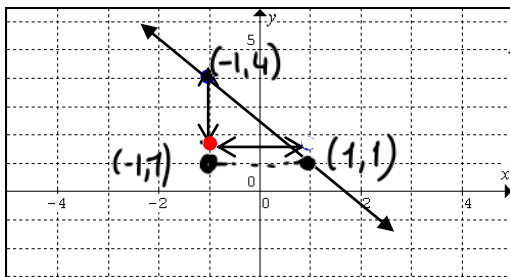
$$\text{slope } \overline{CD} = \frac{8}{2} = 4$$

$$\text{slope } \overline{EF} = \frac{3}{4}$$

$$\text{slope } \overline{GH} = \frac{-5}{3}$$



We don't want to plot points EVERY time we want to know slope. We can create a formula that uses two points to calculate the slope.



The points given here are: (-1, 4) and (1, 1).  
 Pretend there is a point where the two arrows meet.  
 This point is (-1, 1).  
 How can you use these 3 points to find the vertical distance (rise) and the horizontal distance (run)?

vertical  $4 - 1 = 3$   
 horizontal  $1 - (-1) = 2$

- Try to create a formula that you can use to calculate the slope of each of the segments you drew above!

$$\text{slope } (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

### Practice with the Slope Formula #1

Find the slope of a line passing through each of the following pairs of points.

State the answer in simplest form.

1.  $(-9, 8)$  and  $(0, 9)$

$$m = \frac{9 - 8}{0 - (-9)} = \frac{1}{9}$$

2.  $(0, 6)$  and  $(5, -2)$

$$m = \frac{-2 - 6}{5 - 0} = \frac{-8}{5}$$

$3. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (6, 0) & & (0, -6) & \end{matrix}$ $m = \frac{-6-0}{0-6} = \frac{-6}{-6} = 1 \quad \boxed{\text{slope} = 1}$	$4. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (-4, 1) & & (-8, -3) & \end{matrix}$ $m = \frac{-3-1}{-8-(-4)} = \frac{-4}{-8+4} = \frac{-4}{-4} = 1 \quad \boxed{\text{slope} = 1}$
$5. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (-9, 3) & & (-8, -3) & \end{matrix}$ $m = \frac{-3-3}{-8-(-9)} = \frac{-6}{-8+9} = \frac{-6}{1} = -6 \quad \boxed{\text{slope} = -6}$	$6. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (-4, 4) & & (2, -3) & \end{matrix}$ $m = \frac{-3-4}{2-(-4)} = \frac{-7}{2+4} = \frac{-7}{6} \quad \boxed{\text{slope} = -7/6}$
$7. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (5, -4) & & (6, 9) & \end{matrix}$ $m = \frac{9-(-4)}{6-5} = \frac{9+4}{1} = 13 \quad \boxed{\text{slope} = 13}$	$8. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (-8, -5) & & (0, 3) & \end{matrix}$ $m = \frac{3-(-5)}{0-(-8)} = \frac{3+5}{0+8} = \frac{8}{8} = 1 \quad \boxed{\text{slope} = 1}$
$9. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (-1, -9) & & (-6, -2) & \end{matrix}$ $m = \frac{-2-(-9)}{-6-(-1)} = \frac{-2+9}{-6+1} = \frac{7}{-5} = -7/5$	$10. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (-3, 1) & & (-1, -6) & \end{matrix}$ $m = \frac{-6-1}{-1-(-3)} = \frac{-7}{-1+3} = \frac{-7}{2}$
$11. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (11, 17) & & (-8, -18) & \end{matrix}$ $m = \frac{-18-17}{-8-11} = \frac{-35}{-19} = 35/19$	$12. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (-14, 18) & & (8, 0) & \end{matrix}$ $m = \frac{0-18}{8-(-14)} = \frac{-18}{8+14} = \frac{-18}{22} = -9/11$
$13. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (14, -19) & & (-2, -13) & \end{matrix}$ $m = \frac{-13-(-19)}{-2-14} = \frac{-13+19}{-16} = \frac{6}{-16} = -3/8$	$14. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (-2, 14) & & (-9, -17) & \end{matrix}$ $m = \frac{-17-14}{-9-(-2)} = \frac{-31}{-9+2} = \frac{-31}{-7} = 31/7$
$15. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (-16, 5) & & (-5, -5) & \end{matrix}$ $m = \frac{-5-5}{-5-(-16)} = \frac{-10}{-5+16} = \frac{-10}{11}$	$16. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (-17, 7) & & (9, -4) & \end{matrix}$ $m = \frac{-4-7}{9-(-17)} = \frac{-11}{9+17} = \frac{-11}{26}$
$17. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (-49, -86) & & (25, 93) & \end{matrix}$ $m = \frac{93-(-86)}{25-(-49)} = \frac{93+86}{25+49} = \frac{179}{74}$	$18. \quad \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ \swarrow & \searrow & \swarrow & \searrow \\ (-91, -20) & & (-43, 3) & \end{matrix}$ $m = \frac{3-(-20)}{-43-(-91)} = \frac{3+20}{-43+91} = \frac{23}{48}$

### Practice with the Slope Formula #2

1. Calculate the slope of the line given the following two points:

- |                         |                         |
|-------------------------|-------------------------|
| a) (5, 2) and (-1, 8)   | (b) (-8, 1) and (-9, 2) |
| c) (3, 7) and (-5, -9)  | (d) (-4, 0) and (4, 6)  |
| e) (0, 0) and (-2, 10)  | (f) (-6, 24) and (4, 4) |
| g) (8, -7) and (-6, -7) | (h) (-2, 1) and (-1, 3) |

$$a) \quad m = \frac{8-2}{-1-5} = \frac{6}{-6} = -1$$

$$d) \quad m = \frac{6-0}{4+4} = \frac{6}{8} = 3/4$$

$$g) \quad m = \frac{-7+7}{-6+8} = 0$$

$$b) \quad m = \frac{2-1}{-9+8} = \frac{1}{-1} = -1$$

$$c) \quad m = \frac{10-0}{-2-0} = \frac{-10}{-2} = 5$$

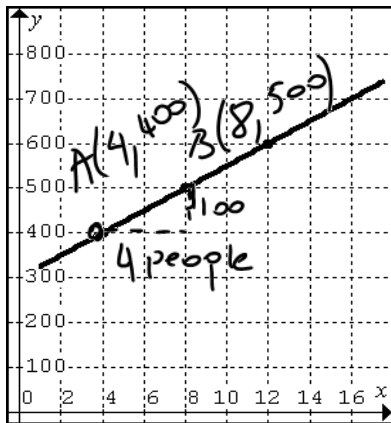
$$h) \quad m = \frac{3-1}{-1+2} = \frac{2}{2} = 1$$

$$e) \quad m = \frac{-9-7}{-5-3} = \frac{-16}{-8} = 2$$

$$f) \quad m = \frac{4-24}{4+6} = \frac{-20}{10} = -2$$

a) The following graph shows the cost of renting a banquet hall. Initially the cost is \$300 just for the hall. There is a per person cost in addition to the initial fee to cover the meal cost.

\$



How much does it cost for each additional person who attends the event? This value is called the **rate of change**, and is a unit rate – in this case cost per person.

$$\text{Unit Rate} = \frac{100}{4} = \$25/\text{person}$$

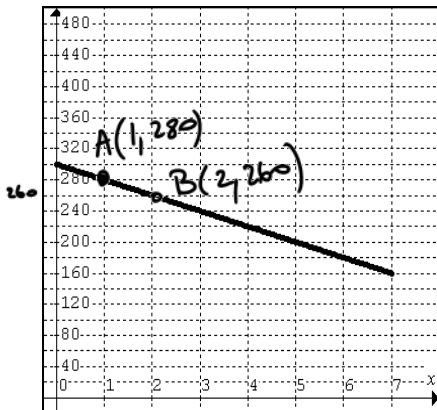
Calculate the slope of this line.

$$\text{slope } \overline{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{500 - 400}{8 - 4} = \frac{100}{4} = 25$$

b) The following graph shows the balance in Jenny's bank account over 7 weeks. She started with \$300 in her account but has been spending her money at a constant rate.

How much did her account decrease by each week? This value is called the rate of change, and in this case is spending per week.

\$ 20



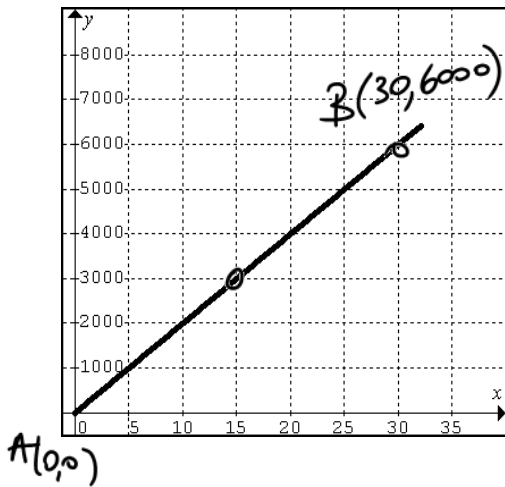
Calculate the slope of this line.

$$\text{slope } \overline{AB} = \frac{260 - 280}{2 - 1} = \frac{-20}{1} = -20$$

c) Molly is an antique hunter. Her father had found a unique gem years ago on the ground (it was free!!). Over time this item became more and more rare. The value for this item increased at a constant rate over the years and now, 30 years later, it is worth \$6000. Each year the value increased by around \$600. Calculate the slope of the line.

How much did this gem increase in value each year? This value is called the rate of change, and in this case is \$ value per year.

Rate of change \$600



Calculate the slope of this line.

$$m_{\overline{AB}} = \frac{6000 - 0}{30 - 0} = \frac{6000}{30} = 500$$