Tables, graphs and equations are all examples of **mathematical models.**

LINEAR MODELS

Represent quantities that increase or decrease by a constant amount over equal intervals

- In a table of values, the first differences are equal.
- The graph is a straight line.
- The equation of the line can be written in the form $y = mx + b$, where m is the slope and b is the vertical intercept (y-intercept) ex $y = 2x + 1$ $C = 10 + 3h$
- The rate of change (slope) is constant
- The vertical intercept represents the initial value of the ***dependent*** variable (y-axis)
- The slope represents the rate of change in the ***dependent*** variable with respect to the ***independent*** variable

KEY WORDS
Constant
Equal
First differences
Straight line
y = mx + b
slope
vertical
constant
initial
rate of change

check if the intervals are constant

EXAMPLE 1 Which models represent linear relations?

a)

Time (s)	Height (m)
0	60
1	55
2	40
3	15

* *Find 1st differences*

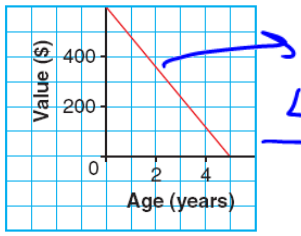
+1: 55 − 60 = −5
+1: 40 − 55 = −15
+1: 15 − 40 = −25
→ changing ∴ NON-LINEAR

b)

Time (h)	Earnings ($)
0	0
5	40
10	80
15	120

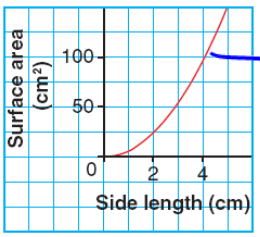
5: 40 − 0 = 40
5: 80 − 40 = 40
5: 120 − 80 = 40
→ constant ∴ Linear

c)



straight line
LINEAR

d)



curve
NON-LINEAR

e) $y = 2x + 5$ LINEAR

f) $y = x^2 + 5$ NON-LINEAR

EXAMPLE 2: A cup of coffee is reheated in a microwave. The temperature, *C* degrees Celsius, of the coffee after *t* seconds can be modelled by the following linear equations. Explain what the numbers in the equations represent. How do the two equations compare to each other?

500 W microwave: $C = 0.5t + 20$ ($y = mx + b$)

1000 W microwave: $C = 1t + 20$

0.5 degree measured at given time
t number of second
20 initial temperature
temperature goes up by 0.5°C per every second
1 temperature goes up by 1°C per sec

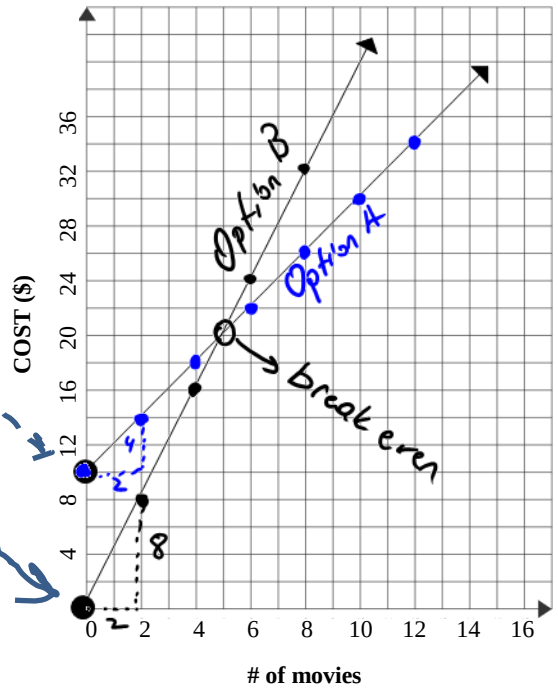
1000W microwave's equation has a higher slope. It would form a steeper graph which tells us the food gets hotter faster.

EXAMPLE 3: Movie Night
There are two payment options for downloading movies from a Web site:

Option A: Pay $10 for a membership and $2 per movie downloaded.
Option B: Pay $4 per movie download.

a) Fill out the chart below then graph each option on the Cartesian plane.

	OPTION A	OPTION B
# of MOVIES	**COST ($)**	**COST ($)**
0	= 10 + 2 x 0 = $10	= 4 x 0 = $0
2	=10+2x2 =$14	= 4x2 =8
4	=10+2x4 =$18	4x4 = 16
6	=10+2x6 =$22	=$24
8	=$26	$32
10	=$30	$40
12	$34	$48



b) Determine the equation for each option in y = mx + b form.
m = slope (rate per movie) b = initial cost (where graph starts on y-axis)

m(Rate) = 4/2 = 2
b (initial cost) = 10
y = 2x + 10 or C = 2m + 10
y = 10 + 2x or C = 10 + 2m

Option B
m(roc) = rise/run = 8/2 = 4 (rate)
b (initial value) = 0
y = 4x + 0 or C = 4m
y = 4x

c) Determine **under which conditions** a person should select Option A and under which conditions a person should select Option B.

If you rent 5 movies either option is ok, less than 5 option B, more than 5 option A.