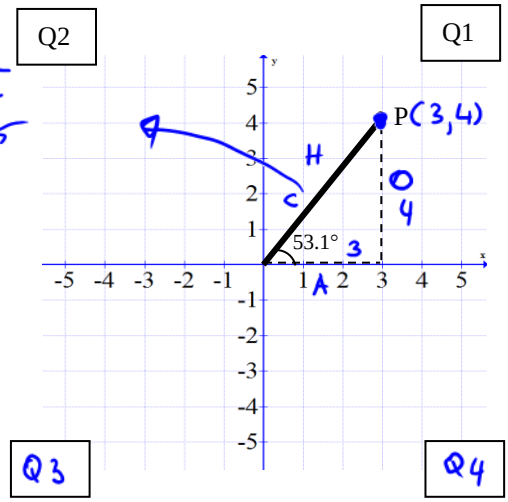
Trigonometric Ratios for Obtuse Triangles

Cartesian coordinate system is divided into **4 quadrants.**

- Label the quadrants on the grid
- Identify the opposite, adjacent and hypotenuse of reference angle 53.1^{O}
- What is the x-coordinate of point P? 3
  What is the y-coordinate of point P? 4
- Write the 3 trigonometric ratios using the triangle

$H = \sqrt{3^2+4^2}$
$H = \sqrt{25} = 5$



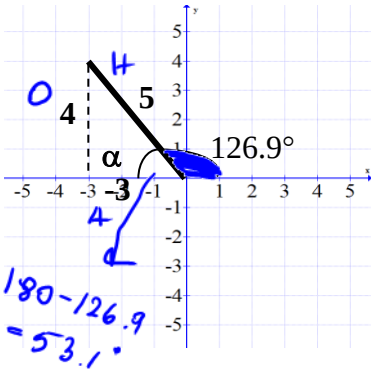
Sin 53.1 = $\frac{O}{H}$ = $\frac{4}{5}$

Cos 53.1 = $\frac{A}{H}$ = $\frac{3}{5}$

Tan 53.1 = $\frac{O}{A}$ = $\frac{4}{3}$

Notice that the length of the adjacent side is the x-coordinate and the length of the opposite side is the y-coordinate. We can use this idea to find the trigonometric ratios of obtuse angles.

angles greater than 90°

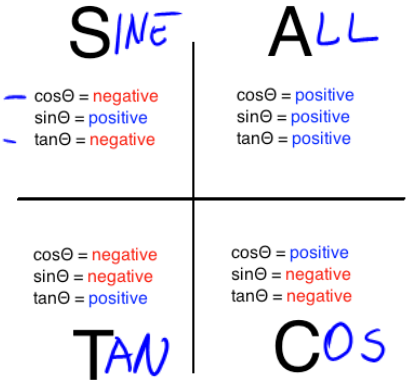


Sin 126.9 = O/H = 4/5

Cos 126.9 = A/H = $\frac{-3}{5}$

Tan 126.9 = O/A = $\frac{4}{-3}$ = $-\frac{4}{3}$

Where are sin, cos and tan positive? This is called the CAST rule:



Notice: For an angle between 0° and 180°,

- If cos or tan are positive the angle is in quadrant 1
- If cos or tan are negative the angle is in quadrant 2
- If sin is positive the angle could be in either quadrant 1 or 2.

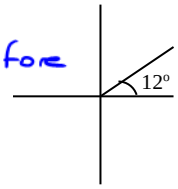
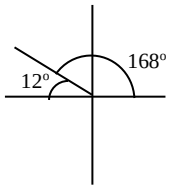
There are always two angles that could give us the same sin ratio.

Q1 Q2 Q2

SINE RATIO (if positive one acute one obtuse, if negative one obtuse)

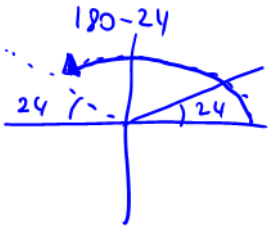
Example: Angle A is between 0^O and 180^O. Determine all measures of angle A in the following case: sin A = 0.2079

Solution: According to CAST rule, sin A is positive in two quadrants: 1 and 2. sin is positive in Q1 & Q2.

ANGLE 1: ACUTE ANGLE in Q1	ANGLE 2: OBTUSE ANGLE in Q2
$\sin^{-1}$ 0.2079 = A A = 12° given sin ratio is '+'ve; therefore we know that there'll be two possible answers. 	Once, we find the acute angle, the obtuse one is rather easy. 180 – 12 = 168 **Check:** sin 168° = 0.2079 

CONCLUSION: A is 12 or 168 degrees.

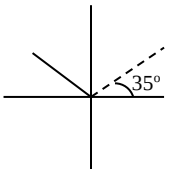
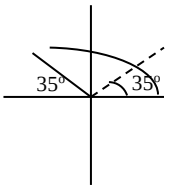
Your turn:

a) sin A = 0.4045	b) sin A = 0.8345
Since sin ratio is +ve in two quadrants. You need to find two angles: acute and obtuse. $\sin^{-1}(0.4045) = A$ $A_1 = 24°$ $A_2 = 180 - 24$ $= 156°$  **Acute**: 24 **Obtuse**: 156°	$\sin^{-1}(0.8345) = A$ $A_1 = 57°$ $A_2 = 180 - 57$ $= 123°$ ∴ Angles are 57° and 123°.

Q1 Q2

COSINE RATIO (if positive only one acute, if negative one obtuse)

Example: Angle A is between 0^O and 180^O. Determine all measures of angle A in the following case: cos A = - 0.8191
Note: We will be dealing with only Q1 and Q2 since angle A is between 0 and 180 degrees.

STEP 1: FIND REFERENCE ANGLE ACUTE ANGLE in Q1	STEP 2: REFLECT THE REFERENCE ANGLE INTO Q2 TO FIND THE OBTUSE ANGLE in Q2
Disregard the sign because it only tells us where the angle is. Since the given cos ratio is -ve. We will seek the angle in Q2. $\cos^{-1}(0.8191) = A$ A = 35 (reference angle)  cos35 is positive. We are interested in the obtuse angle.	 180 – 35 = 145°

CONCLUSION: The obtuse angle A is 145°

TRY

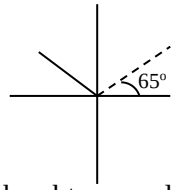
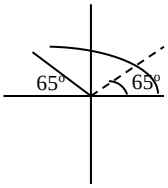
a) cos A = - 0.6345	b) cos A = - 0.3876
$\cos^{-1}(0.6345) = A$ ∴ A is 129°. $A \cong 51°$ 180 - 51 = 129	$\cos^{-1}(-0.3876) = A$ $A \cong 113°$

MAP4C **Date:** _____

Day 3: Trig Ratios -Obtuse Angles **Unit 1: Trigonometry**

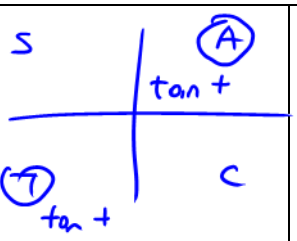
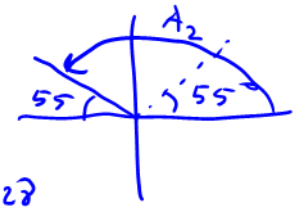
TANGENT RATIO (if positive only one acute, if negative one obtuse)

Example: Angle A is between 0^O and 180^O. Determine all measures of angle A in the following case: tan A = - 2.145
Note: We will be dealing with only Q1 and Q2 since angle A is between 0 and 180 degrees.

STEP 1: FIND REFERENCE ANGLE ACUTE ANGLE in Q1	STEP 2: REFLECT THE REFERENCE ANGLE INTO Q2 TO FIND THE OBTUSE ANGLE in Q2
Disregard the sign because it only tells us where the angle is. Since the given tan ratio is -ve. We will seek the angle in Q2. . $\tan^{-1}(2.145) = A$ A = 65 (reference angle)  tan 65 is positive. We are interested in the obtuse angle.	 $180 - 65 = 115^O$

CONCLUSION: The obtuse angle A is 115°.

YOUR TURN: Angle A is between 0^O and 180^O. Determine all measures of angle A in the following cases:

a) tan A = 1.428 $\tan^{-1}(1.428) = A$ $A = 55°$  	b) tan A = -1.428 $A_2 = 180 - 55$ $= 125°$  Check $\tan 125 = -1.428$

PRACTICE

Angle A is between 0^O and 180^O. Determine all measures of angle A in the following cases:

a) sin A = 0.1358 Angle is in the quadrant(s) 1 & 2 $\sin^{-1}(0.1358) = A$ $A_1 \doteq 8°$ 180 - 8 $A_2 = 172°$ ∴ Angles are 8 & 172°.	b) cos A = - 0.7856 Angle is in the quadrant(s) 2 $\cos^{-1}(0.7856) = A$ $A \doteq 38$ The obtuse angle is 180-38 142°.	c) tan A = -2.1945 Angle is in the quadrant(s) 2 $\tan^{-1}(2.1945) = A$ $66° = A$ The obtuse angle is 180-66 114°.
d) sin A = 0.8135 Angle is in the quadrant(s) 1 & 2 $\sin^{-1}(0.8135) = A_1$ $A_1 = 54°$ $A_2 = 180 - 54$ $= 126°$ ∴ Angles are 54 & 126°.	e) cos A = -0.2487 Angle is in the quadrant(s) 2 $\cos^{-1}(0.2487) = A$ $A = 76°$ $180 - 76 = 104$ ∴ The obtuse angle is 104°.	f) tan A = - 5.3854 Angle is in the quadrant(s) 2 $\tan^{-1}(5.3854) = A_1$ $A_1 = 79°$ $180 - 79 = 101°$ ∴ The obtuse angle is 101°.

Complete: p. 23 #1,2, 4d,5d, 6d, 7, 9, 10