ARITHMETIC SERIES

A **series** is the sum of the terms in a sequence.
An **arithmetic series** is the sum of the terms in an arithmetic sequence.

For example, for the arithmetic sequence 1, 4, 7, 10, … , **the arithmetic series** is $1 + 4 + 7 + 10 + \ldots$ where t_4 represents the 4th term, S_4 represents the sum of the first 4 terms.

The sum of the first n terms of an arithmetic sequence (a series) can be calculated in two ways:

❶ $S_n = \dfrac{n[2a+(n-1)d]}{2}$

❷ $S_n = \dfrac{n(t_1+t_n)}{2}$

Decide which one to use based on the information given.

Ex1. For the given arithmetic series, calculate t_{17} and S_{17}.

$3 + 7 + 11 + \ldots$ $\quad a=3 \quad d=4$

$t_n = a + (n-1)d$

$t_n = 3 + (n-1)4$

$t_n = 3 + 4n - 4$

$\boxed{t_n = 4n - 1}$

$t_{17} = 4(17) - 1$

$\boxed{t_{17} = 67}$

$S_{17} = \dfrac{17(3 + 67)}{2} = \dfrac{17(\overset{35}{\cancel{70}})}{\cancel{2}} = 595$

Ex2. Find the sum of the first 12 terms of the arithmetic series with a = 3 and $t_{12} = 36$.

$S_n = \dfrac{n(t_1 + t_n)}{2} \implies S_{12} = \dfrac{12(3 + 36)}{2} = 6 \cdot 39 = 234$

Ex3. Find the sum of the first 25 terms of the arithmetic series where the 14th term is 102 and terms decrease by 9.

$t_{14} = 102 \quad d = -9 \quad S_{25} = ?$

$t_n = a + (n-1)d$

$102 = a + (14-1)(-9)$

$102 = a + (13)(-9)$

$102 = a - 117$

$\boxed{219 = a} \quad \to t_1$

$t_{25} = 219 + (25-1)(-9)$

$= 219 + (24)(-9)$

$= 219 - 216$

$\boxed{t_{25} = 3}$

$S_{25} = \dfrac{n(t_1 + t_{25})}{2}$

$= \dfrac{25(219 + 3)}{2}$

$= \dfrac{25(\overset{111}{\cancel{222}})}{\cancel{2}}$

$\boxed{S_{25} = 2775}$

Ex4. Calculate the sum of the arithmetic series.

- 4 - 10 - 16 - … - 94 $a = -4$ $d = -10 - (-4) = -6$

$$t_n = a + (n-1)d$$
$$t_n = -4 + (n-1)(-6)$$
$$= -4 - 6n + 6$$
$$\boxed{t_n = -6n + 2}$$

$$-94 = -6n + 2$$
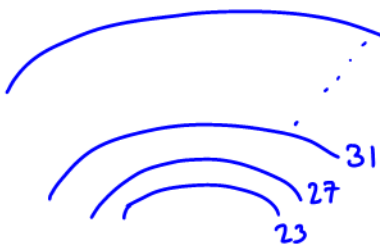$$-96 = -6n$$
$$\boxed{16 = n}$$

$$\boxed{t_{16} = -94}$$
$$\boxed{t_1 = -4}$$

$$S_{16} = \frac{\overset{8}{\cancel{16}}(-94 - 4)}{\cancel{2}}$$
$$= 8(-98)$$
$$= -784$$

Ex5. In an amphitheatre, seats are arranged in 50 semicircular rows facing a domed stage. The first row contains 23 seats, and each row contains 4 more seats than the last. How many seats are there in total?

$$t_n = a + (n-1)d \qquad a = 23 \quad d = 4$$
$$t_n = 23 + (n-1)4$$
$$= 23 + 4n - 4$$
$$\boxed{t_n = 4n + 19}$$

$$t_{50} = 4(50) + 19$$
$$\boxed{t_{50} = 219}$$

$$S_{50} = \frac{\overset{25}{\cancel{50}}(23 + 219)}{\cancel{2}}$$
$$= 25(242)$$
$$\boxed{S_{50} = 6050}$$

Ex6. Samantha deposited $128 into her bank account. Each week, she deposits $7 less than the previous week until she makes her last deposit of $9. Find the total value of her deposits.

$$\underset{128}{\frac{t_1}{\,}} , \underset{121}{\frac{t_2}{\,}} , \underset{114}{\frac{t_3}{\,}} , \quad - - \cdots\cdots \quad \underset{9}{\frac{t_n}{\,}}$$

$a = 128 \quad d = -7$

$$t_n = a + (n-1)d$$
$$9 = 128 + (n-1)(-7)$$
$$9 = 128 - 7n + 7$$
$$7n = 126$$
$$n = 18$$

$$t_{18} = 9$$
$$n = 18$$

$$S_{18} = \frac{\overset{9}{\cancel{18}}(128 + 9)}{\cancel{2}}$$
$$S_{18} = 9(137)$$
$$\boxed{S_{18} = 1233}$$

∴ She deposited $1233 in total.