

COMPARING EXPERIMENTAL AND THEORETICAL PROBABILITY

Theoretical Probability: *mathematical probability that is calculated without actually doing the activity* } $P(\text{event}) = \frac{\# \text{ successful outcomes}}{\text{Total } \# \text{ possibilities}}$

Experimental Probability: *probability observed when performing an experiment* } $P(\text{event}) = \frac{\# \text{ successful trials}}{\text{Total } \# \text{ trials}}$

The theoretical probability of rolling a 2 with a single die is $\frac{1}{6}$. However, if you try it, do you suppose you will roll exactly one 2 in every six trials?

No, reality and theory are not necessarily the same

Suppose that you toss 10 coins at once, repeatedly.

a) How many heads should occur in each toss?

$$\begin{aligned} P(\text{heads}) &= \frac{1}{2} & \therefore \# \text{ expected heads} &= P(\text{heads}) \times \# \text{ trials} \\ & & &= \frac{1}{2} \times 10 \\ & & &= 5 \end{aligned}$$

b) Explain why you will not necessarily see 5 heads every time.

Probabilities are like averages. If the experiment is done enough times the experimental probability gets closer to the theoretical probability

EXAMPLE Darts Anyone?

A modified dart board has equally spaced sectors containing the numbers 1 through 20.

a) What is the theoretical probability of hitting the numbers 18, 19, or 20?

$$P(18, 19, 20) = \frac{3}{20} \quad \begin{array}{l} \leftarrow 3 \text{ successful possibilities} \\ \leftarrow 20 \text{ overall possibilities} \end{array}$$

b) In 75 of the first 80 throws, a dart player hit the numbers 18, 19, or 20. Find the experimental probability of hitting 18, 19, or 20.

$$P(18, 19, 20) = \frac{75}{80} = \frac{15}{16}$$

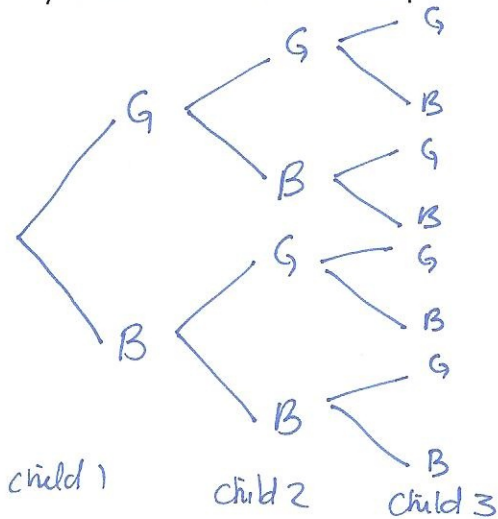
c) Would you expect the experimental probability to approach the theoretical probability if the number of throws increased? Explain.

No, because each outcome is NOT equally likely since the player is actually aiming at 18, 19, and 20 on the board rather than making random shots.

EXAMPLE How Many Girls???

Suppose a couple would like to have three children.

- a) Determine the theoretical probability of having two girls and one boy.

Possible Outcomes

1. GGG
2. GGB
3. GBG
4. GBB
5. BGG
6. BGB
7. BBG
8. BBB

$$P(2G, 1B) = \frac{\# \text{ successes}}{\text{Total \# possibilities}}$$

$$= \frac{3}{8}$$

$$= 38\%$$

- b) Explain how your answer in part a) can help determine the theoretical probability of having two boys and one girl.

The theoretical probability of having 2B and 1G is the same as having 2G and 1B.

- c) Determine the theoretical probability of having at least one girl.

$$P(\text{at least 1G}) = \frac{7}{8} \text{ or } 88\%$$

- d) Determine the theoretical probability of having three girls.

$$P(3 \text{ girls}) = \frac{1}{8} \text{ or } 13\%$$

- e) Suppose the couple hopes to have a boy first and a girl second. What is the likelihood (probability) of that happening?

$$P(BG) = \frac{2}{8}$$

$$= \frac{1}{4} \text{ or } 25\%$$