EVOLUTION OF NUMBERS & NUMBER SETS

The Counting Numbers

We can use numbers to **count:** 1, 2, 3, 4, etc. We humans have been using numbers to count with for thousands of years. It is a very natural thing to do.

- You can have "3 friends"
- a field can have "6 cows"

So we have:

Counting Numbers: {1, 2, 3…}

Zero

The idea of **zero,** though natural to us now, was not natural to early humans…if there is nothing to count, how can you count it?

Example: you can count dogs, but you can't count an empty space:


Two Dogs


Zero Dogs? Zero Cats?

An empty patch of grass is just an empty patch of grass!

But about 3,000 years ago people needed to tell the difference between numbers like 4 and 40. Without the zero they look the same! So they used a "placeholder", a space or special symbol, to show "there are no digits here"

5 2 So, "5 2" meant "502"

The idea of zero had begun, but it wasn't for another thousand years or so that people started thinking of it as an acutal **number.**
But now we can think, "I had 3 oranges, then I ate the 3 oranges, now I have **zero** oranges."

The Whole Numbers

So, let us add zero to the counting numbers to make **a new set of numbers.**
We need a new name, and that name is "Whole Numbers":

Whole Numbers: {0, 1, 2, 3…}

The Natural Numbers (N)

Natural numbers can mean:

- the "counting numbers": {1, 2, 3…}
- or the "whole numbers": {0, 1, 3…}

depending on the subject. The controversy is caused by whether zero is "natural" or not.

Negative Numbers

We can count forward: 1, 2, 3, 4…
When we can backwards we have negative numbers -1, -2, -3, -4…
When a number is less than zero it is simply negative.

Integers Numbers (Z)

If we include the negative numbers with the whole numbers, we have a **new set of numbers** that are called **integers.**

Integers: {…-3, -2, -1, 0, 1, 2, 3…}

The integers include zero, the counting numbers, and the negative of the counting numbers,

Rational Numbers (Q)

Any number that can be written as a fraction is called a **Rational Number.**

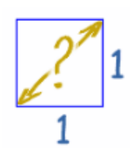
So, if "p" and "q" are integers, then $^{p}/_{q} = {}^{3}/_{2}$ is a rational number.

The only time this does not work is when q is **zero.**

Rational numbers include:

- all the **integers**
- and all **fractions**

IRRATIONAL NUMBERS



If you draw a square (of size "1"), what is the distance across the diagonal?
You know that the is the square root of 2, which is 1.4142135623730950…(etc)
But it is not a number like 3, or five-thirds. So it is not a **rational number**. We call them **Irrational Numbers.** Some examples are $\sqrt{2}$, π (Pi)

You need **irrational numbers to:**

- find the diagonal distance across some squares,
- to work out lots of calculations with circles (using π)

We really should include irrational numbers. Thus, we need to introduce a new set of numbers…

REAL NUMBERS (R)

Real numbers include:

- the rational numbers, and
- the irrational numbers

A Real Number can be thought of as any number.

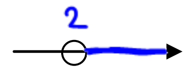
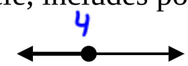
SUMMARY

N

Type of Number	Quick Description
Counting Numbers	{1, 2, 3, ...}
Whole Numbers	{0, 1, 2, 3, ...}
Integers Z	{..., -3, -2, -1, 0, 1, 2, 3, ...}
Rational Numbers Q	p/q : p and q are integers, q is not zero
Irrational Numbers	Not Rational
Real Numbers R	Rationals and Irrationals

This lesson was inspired by www.mathisfun.com

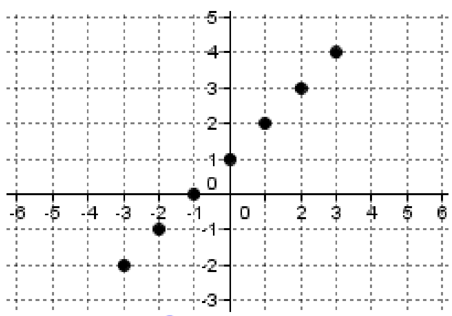
SYMBOLS

> **greater than** e.g. x > 0, x is greater than 0	o open circle, does not include point. e.g. x > 2 
< **less than** e.g. x < 0, x is less than 0	● closed circle, includes point e.g. a ≤ 4 
≥ **greater than or equal to** e.g. x ≥ 0 x is greater than or equal to 0	{ } set
≤ **less than or equal to** e.g. x ≤ 0, x is less than or equal to 0	| such that I is an element of, or is in the set of
Notation Format Good form: 0 ≤ x ≤ 8 (x is between 0 and 8) Bad form: anything else Ex. x ≥ 0 and x ≤ 8	

How to state Domain - D: {x| x∈R} x **such that** x can be any value in the **Real** numbers. "∈" is the symbol meaning "in the set of"	**How to state Range - R**: {y| y≥5, y∈R} It means y such that y is greater than or equal to 5, where y is a Real number.

1. State the **domain** and **range** of each relation from its graph..

a)

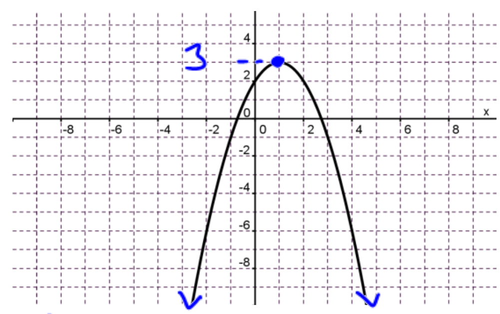


D={x∈Z | -3 ≤ x ≤ 3} or D={-3,-2,-1,0,1,2,3}

R={y∈Z | -2 ≤ y ≤ 4} or R={-2,-1,0,1,2,3,4}

FUNCTION

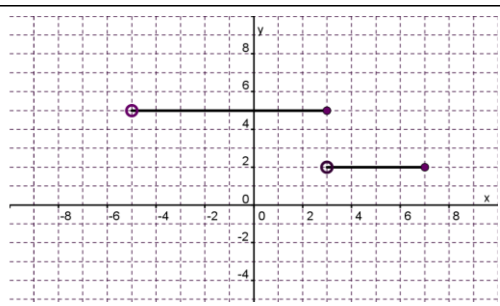
b)



D= { x ∈ R } x is element of Real Numbers

R= { y∈R | y ≤ 3 }

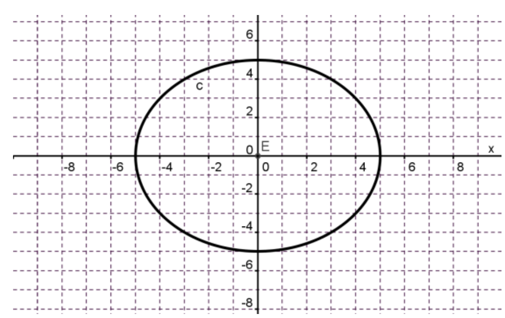
c)



D= { x∈R | -5 < x ≤ 7 }

R= { 2 , 5 }

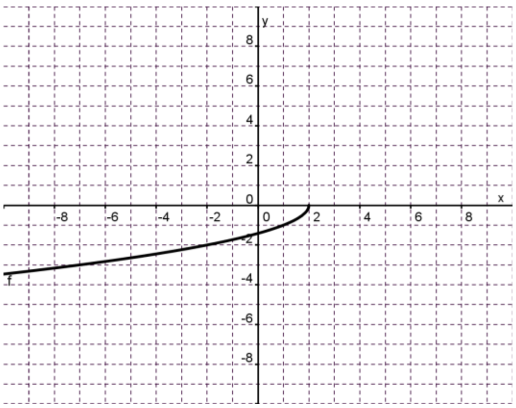
d)



D= { x∈R | -5 ≤ x ≤ 5 }

R= { y ∈ R | -5 ≤ y ≤ 5 }

e)

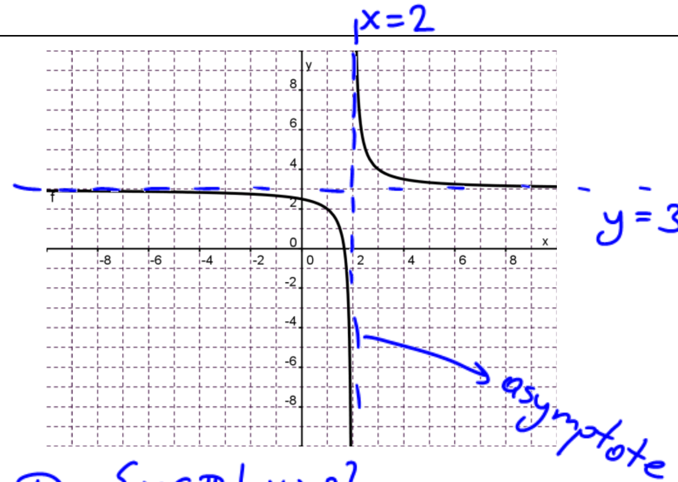


$D = \{x \in \mathbb{R} \mid x \leq 2\}$

$R = \{y \in \mathbb{R} \mid y \leq 0\}$

FUNCTION

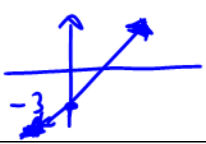
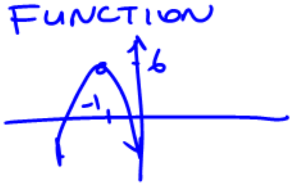
f)



$D = \{x \in \mathbb{R} \mid x \neq 2\}$

$R = \{y \in \mathbb{R} \mid y \neq 3\}$

2) Determining Domain And Range From The Function Equation

a) $f(x) = 2x - 3$ LINEAR FUNCTION 	$D = \{x \in \mathbb{R}\}$ x could be any Real number $R = \{y \in \mathbb{R}\}$ y such that y is an element of any Real number
b) $g(x) = -3(x+1)^2 + 6$ QUADRATIC FUNCTION 	$D = \{x \in \mathbb{R}\}$ $R = \{y \in \mathbb{R} \mid y \leq 6\}$
c) $h(x) = \sqrt{2 - x}$ SQUARE ROOT FUNCTION	$2 - x$ cannot be negative $2 - x \geq 0$ $D = \{x \in \mathbb{R} \mid x \leq 2\}$ $2 \geq x$ $x \leq 2$ $R = \{y \in \mathbb{R} \mid y \geq 0\}$