MPM1D **Date:**

Day 4: Fractions – Reducing, Converting, Multiplying & Dividing **Unit 1: Number Sense**

A fraction is made up of two parts. The top of the fraction is called the NUMERATOR and the bottom of the fraction is called the DENOMINATOR.

If the numerator is greater than the denominator, this is improper fraction (Ex: $\frac{8}{3}$). These types of fractions can also be written as a whole number and a fraction. This is a mixed number (Ex: $2\frac{2}{3}$).

1) REDUCING FRACTIONS TO LOWEST TERMS

RULE: When using fractions, your solutions must always be given in lowest terms.
Step 1: Reduce the amount of signs to ONE.
Step 2: List the factors (numbers that divide evenly into) of the numerator and denominator.
Step 3: Find the Greatest Common Factor (GCF) of (the greatest number that divides evenly into) the numerator and denominator.
Step 4: Divide both the numerator and denominator by GCF.
Step 5: Ensure that you have ONLY one sign that is place next to the numerator or the fraction.

Ex 1: $\frac{-9}{12} = \frac{9\div3}{12\div3} = \frac{3}{4}$
The factors of 9 are: { 1, 3, 9 }
The factors of 12 are: { 1, 2, 3, 4, 6, 12 }
The GCF is 3. Simply divide the numerator and denominator by this number. These two fractions are also known as **equivalent fractions**.

Ex 2: $\frac{-27}{-45} = \frac{27\div9}{45\div9} = \frac{3}{5}$
The factors of 27 are { 1, 3, 9, 27 }
The factors of 45 are { 1, 3, 5, 9, 15, 45 }
The GCF is 9.

Try these:

a) $\frac{16}{20} = \frac{16\div4}{20\div4} = \frac{4}{5}$ GCF = 4

b) $\frac{20}{-100} = \frac{-20\div20}{100\div20} = \frac{-1}{5}$ GCF = 20

c) $-\frac{-24}{36} = \frac{24\div12}{36\div12} = \frac{2}{3}$ GCF = 12

2) CONVERTING IMPROPER FRACTION TO MIXED NUMBER

Ex 3: Convert $\frac{13}{4}$ into a mixed number. *Simply, how many groups of 4 go into 13?*

We need to figure out two numbers. Firstly, the whole number and secondly, the numerator. We already know the denominator which is 4.

IMPORTANT: Reduce the amount of signs to one before any conversion.

Step 1: Divide 13 by 4. The answer is 3.25. Thus, there are 3 groups of 4 that go into 15. Three is the whole number of our mixed number.

Step 2: 3 groups of 4 make 12 (3 times 4). |||| |||| |||| ||||

Step 3: Subtract 12 from 13. The answer is 1. When 13 is divided into groups of 4. There will be 1 remaining. One is the numerator of the mixed number.

$3\frac{1}{4}$

Try these: a) $\frac{7}{2} = 3\frac{1}{2}$ b) $\frac{-17}{8} = -2\frac{1}{8}$ c) $-\frac{-34}{-7} = -4\frac{6}{7}$

3) CONVERTING MIXED NUMBERS INTO IMPROPER FRACTIONS

This is the opposite of what we did in the previous section.

$3\frac{2}{7}$ is telling me that there are 3 groups of 7 bundled together and 2 remaining. Therefore, the total must be 3 x 7 + 2 which is 23. Thus, the answer is 23/7.

IMPORTANT: DISREGARD THE SIGN UNTIL THE CONVERSION IS DONE

To convert mixed numbers to improper fractions: $w\frac{n}{d} = \frac{w \times d + n}{d}$ or $-w\frac{n}{d} = -\left(\frac{w \times d + n}{d}\right)$

Example 4: $2\frac{3}{5} = \frac{2\times5+3}{5} = \frac{10+3}{5} = \frac{13}{5}$

Example 5: $-2\frac{3}{5} = -\frac{2\times5+3}{5} = -\frac{10+3}{5} = -\frac{13}{5}$

Try these:

a. $4\frac{7}{8} = \frac{4\times8+7}{8} = \frac{32}{8} = \frac{39}{8}$

b. $5\frac{11}{12} = \frac{5\times12+11}{12} = \frac{71}{12}$

c. $-3\frac{1}{5} = -\left(\frac{3\times5+1}{5}\right) = -\frac{16}{5}$

4) MULTIPLYING FRACTIONS

Step 1: Convert any mixed number into improper fraction.
Step 2: Multiply the numerators with each other. This is your numerator.
Step 3: Multiply the denominators with each other. This is your denominator.
Step 4: Reduce the final answer.

Ex 6: $\frac{1}{7}\times\frac{2}{3} = \frac{1\times2}{7\times3} = \frac{2}{21}$

Ex 7: $\left(1\frac{3}{4}\right)^3 = \left(\frac{1\times4+3}{4}\right)^3 = \left(\frac{7}{4}\right)^3 = \left(\frac{7}{4}\right)\left(\frac{7}{4}\right)\left(\frac{7}{4}\right) = \frac{343}{64}$

5) DIVIDING FRACTIONS

Step 1: Convert any mixed number into improper fraction.
Step 2: Reciprocate the second fraction. Simply, switch the numerator with the denominator.
Step 3: Division becomes multiplication. Follow the rules for multiplication.

Ex 8: $\frac{1}{2}\div\frac{8}{3} = \frac{1}{2}\times\frac{3}{8} = \frac{1\times3}{2\times8} = \frac{3}{16}$

Ex 9: $\frac{-1}{2}\div\frac{5}{-4} = -\frac{1}{2}\times\frac{-4}{5} = \frac{(-1)(-4)}{2\cdot5} = \frac{4}{10}$ GCF = 2 $\frac{4\div2}{10\div2} = \frac{2}{5}$

Try these:

a) $2\frac{2}{5}\times\frac{-1}{4} = \frac{2\cdot5+2}{5}\times\frac{-1}{4} = \frac{10+2}{5}\cdot\frac{-1}{4} = \frac{12}{5}\cdot\frac{-1}{4} = \frac{-12}{20} = \boxed{\frac{-3}{5}}$

b) $3\frac{2}{5}\div-2\frac{1}{4} = \frac{3\cdot5+2}{5}\div\frac{-2\cdot4+1}{4} = \frac{17}{5}\div\frac{-9}{4} = \frac{17}{5}\cdot\frac{-4}{9} = \frac{-68}{45}$

c) $\left(\frac{-3}{5}\right)^2\div\frac{27}{10} = \left(\frac{-3}{5}\right)\left(\frac{-3}{5}\right)\div\frac{27}{10} = \frac{9}{25}\times\frac{10}{27} = \frac{90}{675} = \frac{2}{15}$