## MCR3U1 Day 4: Max Profit, Revenue and Break Even

The cost function C(x) is the total cost of making x items. If the cost per item is fixed, it is equal to the cost per item (c) times the number of items produced (x), or

 $C(x) = c \bullet x$ 

The price function  $\mathbf{p}(\mathbf{x})$  – also called the demand function – describes how price affects the number of items sold. Normally, when the price increases, customers will not demand as many items, and so x will decrease. To sell more items, the price usually has to decrease.

The **revenue function**  $\mathbf{R}(\mathbf{x})$  is the income from sales. It is equal to the price times the number sold, or

 $R(x) = x \bullet p(x)$ 

The **profit function** P(x) is the money that is left over from the revenue (income) after the costs (expenses) have been subtracted. In other words,

$$P(x) = R(x) - C(x) \qquad \bigcirc \checkmark$$

## **BREAK EVEN:** Revenue = Cost

The **break-even point** occurs when the total revenue equals the total cost - or, in other words, when the profit is zero. To solve for a break-even quantity, set P(x) = 0 and solve for x using factored form or the quadratic formula.

Warm Up: You are the sole owner of a denim store downtown, Toronto. Last week, you sold 200 pairs of jeans priced at \$36. You buy the pants from a local manufacturer located in Montreal, Quebec. To operate your business, it costs you average \$20 per pants.

a) Calculate your total revenue for the last week.

b) Determine the total profit.

6) Profit = Revenue - Cost = 7200 - 200.20a) Revenue = Price & amount = 36 × 200 7200 - 4000 - 7200 **\$**32.00

Ever since you took grade 11 math course, you have been wondering if you can apply maximizing total revenue and profit concept in your business to make more money. When you set your price for \$36, you sell average of 200 pairs a week. After doing a mini survey, you find out that for each \$2 increase in price, you sell 5 fewer pants. To operate your business, it costs you average \$20 per pants. of us de change

a) What price will maximize your total revenue?
Revenue = Price x Amount rice charge (once, twice, x omount of time)
R(x) = (36 + 2.7)(200 - 57) Revenue (x,y) reve man
Stepl: Findics "x" coordinate of vertex
36 + 2x = 0 200 - 5x = 0
2x = -3b $200 = 5x$ $-18$ $-18$
x = -18 $40 = x$ Change
X = -18+40 / Revenue = Price · Amant
$= (36+2\cdot 11)(200-5\cdot 11)$
= (58)(145)
= \$840
. When you set the price \$58, you will max the revense \$8410

**Chapter 3: Quadratic Relations** 

Date:

b) What price will maximize your total profit? What is the total profit at this price?

$$\begin{array}{l} \Pr \text{rofit} = \text{Revenue} - \text{Cost} \\ P(x) = R(x) - C(x) \\ = (3b + 2x)(200 - 5x) - 20(200 - 5x) \quad \text{GcF} = (200 - 5x) \\ = (200 - 5x)(36 + 2x - 20) \quad \text{Profit} \\ P(x) = (200 - 5x)(16 + 2x) \quad \text{yf} \quad (x, y) \\ 200 - 5x = 0 \quad 16 + 2x = 0 \\ 200 - 5x = 0 \quad 16 + 2x = 0 \\ 200 - 5x = 0 \quad 16 + 2x = 0 \\ 200 - 5x = 0 \quad 16 + 2x = 0 \\ (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8} \\ \hline (40 - x) \quad x = \frac{7}{8} \quad 2x = \frac{7}{8}$$

c) Calculate break-even point? What is the price at these points?

$$P(x) = 200 - 5x7(16 + 2x)$$

$$200 - 5x = 2$$

$$|6 + 2x = 0$$

$$X = -8$$

$$P(iw = 36 + 2x)$$

$$-36 + 2(40)$$

$$= -36 + 2(40)$$

$$= -36 - 16$$

$$= -36 - 16$$

$$= -16$$

Profit 1 -8 40 Price Chay

. When you set the price \$20 or \$116 you do break even.

1. When priced at \$40 each, a toy company sells 5000 toys. The manufacturer estimates that each \$1 increase in price will decrease sales by 100 units. Find the unit price of a toy that will maximize the total revenue.

Sheet "het "x" rep number of price change  
Rev = Price x Amount Sold  

$$R(x) = (40 + 1x) (5000 - 100x)$$
  
 $40 + x = 0$   
 $(x = -40)$   
Sheet Averaging 2602  
 $x = -\frac{10}{2}$  We just found the # of price change  
 $x = -\frac{10}{2}$  We just found the # of price change  
 $x = \frac{10}{2}$  We just found the # of price change  
 $x = \frac{10}{2}$  Price = 40 + x  
 $= 40 + 5$   
 $= 45$   
 $\therefore$  The price must be <sup>\$</sup>45 to maximize the revence.

2. The city transit system carries 24800 bus riders per day for a fare of \$1.85. The city hopes to reduce car pollution by getting more people to ride the bus, while maximizing the transit system's revenue at the same time. A survey indicates that the number of riders will increase by 800 for every \$0.05 decrease in the fare. What fare will produce the **greatest revenue**?

Revenue = Price · Riders det "x" rep # of change in price  

$$R(x) = (1.85 - 0.05x)(24.800 + 800x)$$
  
We need to find the "x" coordinate of  
the vertex to calculate the price.  
 $1.85 = 0.05x = 0$  24.800 + 800x = 0  
 $1.85 = 0.05x = 0$  24.800 + 800x = 0  
 $1.85 = 0.05x = 800x = 24.800$   
 $X = 3T$   
Average 2005  
 $x = -31 + 37$   
 $x = -31 +$ 

3. The Thunderbirds professional indoor soccer team has 900 season ticket holders. The management of the team wants to increase the current price of \$400. A survey indicated that for every \$20 increase in price, the team will lose 15 season ticket holders. What price would maximize revenue from season ticket holders? What is the **maximum revenue** the team could receive?



## \$800 will maximize the revenue which is \$489,000.

4. The school council sells sweatshirts to raise funds. The students sell 500 sweatshirts a year at \$45 each. They are planning to decrease the price to generate more sales. An informal survey was taken showing that for every \$1 decrease in price they can expect to sell an additional 20 sweatshirts. If the survey results are correct, what price would maximize revenue from sweatshirt sales? How many sweatshirts must be sold? What would be the **maximum revenue** generated?

5. An amusement park charges \$8 admission and averages 2000 visitors per day. A survey shows that for each \$1 increase in the admission price, 100 fewer people would visit the park.

- (Na) Determine what price the amusement park should charge to maximize revenue.
- **b**) At what price(s) will the revenue be equal to \$0?

c) Find the price(s) that would generate revenue of \$11500.



## **Practice: Maximizing PROFIT – BREAK EVEN Problems**

1. Research for a given orchard has shown that, if 100 pear trees are planted, then the annual revenue is \$90 per tree. If more trees are planted, they have less room to grow and generate fewer pears per tree. As a result, the annual revenue per tree is reduced by \$0.70 for each additional tree planted. No matter how many trees are planted, the cost of maintaining each tree is \$7.40 per year. How many pear tree should be planted to maximize the **profit** from the orchard for one year?

$R(x) = Price \cdot Amount$ = (90-0.70x)(100+x)	C(x) = Cost Price · Amount = 7.40(100+x)
P(x) = (90-0.70x)(100+x) - 7.400 = (100+x)(90-0.70x) - 7.400 = (100+x)(90-0.70x) - 7.400 = 0.70x)	$\begin{array}{c} O(100+x) & GCF=(100+x) \\ O & Profit \\ 0 & 1 \\ \end{array}$
$\begin{array}{c} 1004x=0 \\  X=-100  \\ \hline x=100 $	) -100 117 # of extra plonts
Vertex (x,y)	
$Y = \frac{-100 + 118}{2}$ $[Y = 9]$	
: 109 pear trees must	be planted. to maximize

the profit.

2. The demand function for a new product is p(x) = -5x + 39, where p represents the selling price of the product and x is the number sold in thousands. The cost function is C(x) = 4x + 30.
a) Find the value of x that will maximize the profit.

b) Find the break-even quantities.  
a) 
$$Profit = Revenue - Cost$$
  
 $R(x) = x \cdot (-5x+39)$   
 $P(x) = x(-5x+39) - (4x+30)$   
 $= -5x^{2}+39x - 4x - 30$   
 $P(x) = -5x^{2}+35x - 30$  we need to find the "x" of vertex  
 $= -5(x^{2}-7x) - 30$   $-7+2 = -3.5$   
 $(-3.5)^{2} = (2.25 + x)$   
 $= -5(x^{2}-7x) - 30$   $-7+2 = -3.5$   
 $(-3.5)^{2} = (2.25 + x)$   
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 $(-3.5)^{2} = (2.25 + x)$   
 $= -5(x^{2}-7x) - 30$   $-7+2 = -3.5$   
 $(-3.5)^{2} = (2.25 + x)$   
 $= -5(x^{2}-7x) - 30$   $-7+2 = -3.5$   
 $(-5.5)^{2} = (2.25 + x)$   
 $= -5(x^{2}-7x) - 30$   $-7+2 = -3.5$   
 $(-5.5)^{2} = (2.25 + x)$   
 $= -5(x^{2}-7x) - 30$   $-7+2 = -3.5$   
 $0 = -5x^{2}+35x - 30$  First GCF = -5  
 $0 = -5(x^{2}-7x) + 6$   $-\frac{M|A||P|}{6||7||-1|, 76}$   
 $x-1=0$   $x-b=0$   
 $|X=1|$   $(X=4)$ 

. Break-even quantities are 1000 and 6000 units.

3. A car rental agency has 150 cars. The owner finds that, at a price of \$48 per day, he can rent all the cars.  
For each \$21 increases in price, the demand is less and 4 fewer cars are rented. For each car that is rented,  
there are routine maintenance costs of \$55 per day.  
a) What rental charge will maximize profit?  
b) Find the break-even points for the profit as well as the price at these points.  
The demand of the profit as well as the price at these points.  
The first = Revene - Cost 
$$R(x) = (47+2x)(150-4x)$$
  
 $= (48+2x)(150-4x) - 5(150-4x)$   
 $= (48+2x)(150-4x) - 5(150-4x)$   
 $= (48+2x)(150-4x)$   
 $= (48+2x)(150-4x)$   
 $P(x) = (43+2x)(150-4x)$   
 $P(x) = (43$ 

4. An auditorium has seats for 1200 people. For the past several days, the auditorium has been filled to capacity for each show. Tickets currently cost \$5.00 and the owner wants to increase the ticket prices. He estimates that for each \$0.50 increase in price, 100 fewer people will attend \$ 3.57
a) What ticket price will maximize the **profit**?

b) Fin the break-even points for the profit as well as the price at these points.

het "x" (cp the number of price change  
Profit = Revenue - Cost 
$$R(x) = Price \cdot Amount$$
  
 $= (5 + 0.50x)(1000 - 100x) - 3.5(1200 - 100x)$   
 $R(x) = (5 + 0.50x)(1200 - 100x) - 3.5(1200 - 100x)$   
 $R(x) = (1200 - 100x)(5 + 0.50x - 3.5)$   
 $R(x) = (1200 - 100x)(1.5 + 0.50x)$   
 $R(x) = (1200 - 100x)(1.5 + 0.50x)$