

MULTIPLYING AND DIVIDING RATIONAL NUMBERS

Prime factors

To multiply or divide rational *expressions* we use the same techniques as with rational *numbers*.

$$\begin{array}{r|l} 20 & 2 \\ 10 & 2 \\ 5 & 5 \\ 1 & \end{array} \quad \begin{array}{r|l} 6 & 2 \\ 3 & 3 \\ 1 & \end{array}$$

Ex1. $\frac{3}{5} \times \frac{2}{7} = \frac{3 \cdot 2}{5 \cdot 7} = \frac{6}{35}$

Ex2. $\frac{9}{5} \times \frac{20}{6} = \frac{9 \cdot 20}{5 \cdot 6} = \frac{3 \cdot \cancel{3} \cdot \cancel{2} \cdot 2 \cdot \cancel{5}}{\cancel{5} \cdot \cancel{2} \cdot 3} = 6$

Ex3. $\frac{3}{5} \div \frac{9}{2} = \frac{3}{5} \times \frac{2}{9} = \frac{3 \cdot 2}{5 \cdot 9} = \frac{\cancel{3} \cdot 2}{5 \cdot \cancel{3} \cdot 3} = \frac{2}{15}$

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

For rational expressions $\frac{P}{Q}$ and $\frac{R}{S}$:

$$\frac{P}{Q} \times \frac{R}{S} = \frac{PR}{QS} \text{ with restrictions } Q \neq 0, S \neq 0$$

$$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \times \frac{S}{R} = \frac{PS}{QR} \text{ with restrictions } Q \neq 0, S \neq 0, R \neq 0$$

Ex1. $\frac{x+1}{x^2-x-6} \times \frac{x^2-7x+12}{x^2-16}$

Restrictions

1) Factor $\frac{(x+1)}{(x+2)(x-3)} \cdot \frac{(x-3)(x-4)}{(x-4)(x+4)}$

2) Restrictions
 $x \neq -2, x \neq 4$
 $x \neq 3, x \neq 4$

2) Multiply = $\frac{(x+1)\cancel{(x-3)}\cancel{(x-4)}}{(x+2)\cancel{(x-3)}\cancel{(x-4)}(x+4)}$
 $= \frac{(x+1)}{(x+2)(x+4)}$ $x \neq -4, -2, 3 \text{ and } 4$

Ex2. $\frac{2x-4}{x^2+9x+20} \div \frac{x^2+x-6}{x^2+7x+12}$

1) Factor $\frac{2(x-2)}{(x+4)(x+5)} \div \frac{(x-1)(x+6)}{(x+3)(x+4)}$

$x \neq -4, x \neq -3$
 $x \neq 5$

flip = $\frac{2(x-2)}{(x+4)(x+5)} \times \frac{(x+3)(x+4)}{(x-1)(x+6)}$

$x \neq 1$
 $x \neq -6$

simpl. = $\frac{2(x-2)(x+3)\cancel{(x+4)}}{\cancel{(x+4)}(x+5)(x-1)(x+6)}$

= $\frac{2(x-2)(x+3)}{(x+5)(x-1)}$ $x \neq -6, -5, -4, -3 \text{ and } 1$

SUMMARY

- Factor numerators and denominators
- Note restrictions on the variables (denominators, and numerators of divisors)
- If applicable, rewrite division as multiplication by the reciprocal
- Write as a single rational expression
- Reduce by cancelling common factors
- Write the simplified rational expression
- Formally state all restrictions on the domain