MAP4C **Date: ____________**

Day 4: Optimizing Perimeter and Area **Unit 2: Measurement & Geometry**

Optimize: Determine the best solution while adhering to given constraints

Concept 1: Maximum Deck Size – *MAXIMIZING THE AREA*

Complete the table below:

Rectangle	Length (m)	Width (m)	Perimeter (m)	Area (m^2)
A	15	5	2(15+5) = 40	15 x 5 = 75
B	12	8	2(12+8) = 40	12x8 = 96
C	10	10	2(10+10) = 40	10x10 = 100
D	18	2	2(18+2) = 40	18 x 2 = 36
E	9	11	2(9+11) = 40	9x11 = 99

OBSERVATION:

The rectangles above have the same perimeter.

Rectangle C has the greatest area.

CONCLUSION:

When enclosing **FOUR SIDES**, the maximum rectangular area for a fixed perimeter is obtained by forming a square.

Solved Example: Find the maximum area for a rectangular four-sided area you can enclose with a 60 m rope at a beach. [Hint: Recall that the four-sided shape will be a SQUARE.]

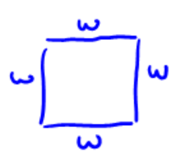
Solution: The square will provide the maximum area. We need to form a square that has a 60m perimeter.
1 side of the square = 60 / 4 = 15
The enclosed area will by 15 by 15; thus, its area is 15 x 15 = 225 m^2.

Try these: Find the dimensions (integer values) of the largest rectangular area that can be made given the following perimeters:

a) 48m 48m of fencing

w = 48 ÷ 4

w = 12m



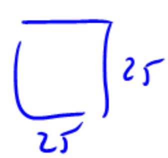
∴ It's 12m by 12m

Area is 144 m^2

b) 100m

w = 100 ÷ 4

= 25



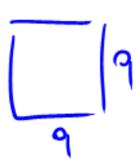
∴ It's 25m by 25m

625 m^2 area.

c) 36 m

w = 36 ÷ 4

$\boxed{w = 9m}$



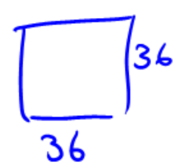
∴ It's 9m by 9m

Area is 81 m^2

d) 144 m

w = 144 ÷ 4

= 36m



∴ It's 36m by 36m

Concept 2: Minimum Perimeter for a Fixed Area – *MINIMIZING THE PERIMETER*

Complete the table below:

Rectangle	Length (m)	Width (m)	Area (m^2)	Perimeter (m)
A	2	18	2 x 18 = 36	2(2 + 18) = 40
B	3	12	3 x 12 = 36	2(3 + 12) = 30
C	4	9	4 x 9 = 36	2(4 + 9) = 26
D	6	6	6 x 6 = 36	2(6 + 6) = 24
E	9	4	9 x 4 = 36	2(9 + 4) = 26

OBSERVATION:
The rectangles above have the same area.
Rectangle D has the least perimeter area.

CONCLUSION:
When enclosing **FOUR SIDES**, the minimum perimeter for a given rectangular area is obtained by forming a square

Solved example: Find the dimensions of the rectangle that will provide the least perimeter if the area is 81 m^2.
Solution: The square will provide the least perimeter. We know the area of this square which is 81.
One side of the square is = $\sqrt{81} = 9$
Therefore, it is a 9 by 9 square that will give the least perimeter.

Try these: Find the dimensions (integer values) of the smallest rectangular perimeter that can be made given the following areas:

a) $36m^2$

$w^2 = 36$

$\sqrt{w^2} = \sqrt{36}$

$w = 6$ $\therefore$ It's 6m by 6m

b) 100 m^2

$w^2 = 100$

$\sqrt{w^2} = \sqrt{100}$

$w = 10$

$\therefore$ It's 10m by 10m.

c) 144 m^2

$w^2 = 144$

$\sqrt{w^2} = \sqrt{144}$

$w = 12$ $\therefore$ It's 12m by 12m

d) 256 m^2

$w^2 = 256$

$\sqrt{w^2} = \sqrt{256}$

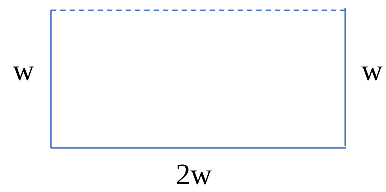
$w = 16$ $\therefore$ It's 16m by 16m

Concept 3: MAXIMUM 3-SIDED AREA

CONCLUSION:

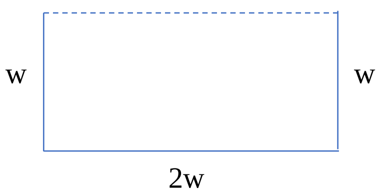
When enclosing **THREE SIDES**, the **maximum area** for a given rectangular area is obtained by forming a rectangle whose length is twice its width.

Let "w" represent the width "2w" represent the length.



Solved example: Your manager asked you to create a three-sided rectangular area at the beach with a 100 m rope that will have the greatest area. What are the dimensions?

Solution: Since this is a three-sided enclosing, we will use the following figure:



$w + 2w + w = 4w$

$4w$ is the perimeter because that is how much rope we have

$4w = 100$ divide both sides by 4

$4w / 4 = 100 / 4$

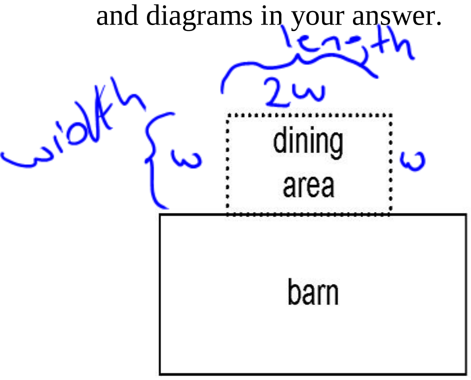
$w = 25$

width = 25 length = 50

Therefore; it is 25 by 50 rectangle with its three sides enclosed.

Your turn:

Organizers of an outdoor music festival want to enclose a rectangular dining area against a large barn. They have 80 m of rope to use as a fence. Determine the length and width of a fence that will provide the maximum dining area. Use words and diagrams in your answer.



$w + 2w + w = 80$

$4w = 80$

$\div 4 \quad \div 4$

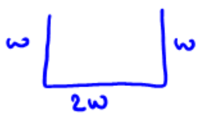
$w = 20$

∴ It'll be 20m by 40m.

PRACTICE

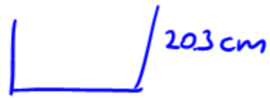
1. An inbox tray has 3 walls and an open side on one of the longer sides. Determine the maximum area of the tray if all three walls total to a length of 812 mm.

Step1



$4w = 812$
$\div 4 \quad \div 4$
$w = 203$ mm
$= 20.3$ cm

Step2



$2 \times 203 = 406$ mm

$A = 40.6 \times 203$
$= 824.18 mm^2$

$\therefore$ Area is $82418 mm^2$
OR $824.18 cm^2$

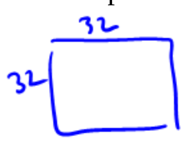
2. The perimeter of a rectangular piece of cardboard is 46 centimetres. Determine the dimensions that maximize the area.

$4w = 46$
$\div 4 \quad \div 4$
$w = 11.5$

$\therefore$ It is 11.5 cm by 11.5 cm.

3. The maximum area of a fenced in pool deck is 1024 m^2. Determine the length of fencing that is required.

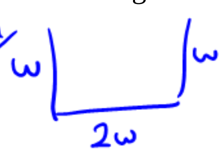
$w^2 = 1024$
$\sqrt{w^2} = \sqrt{1024}$
$w = 32$



$\therefore$ You need $4 \times 32 = 128$ m of fencing.

4. Three sides of a lookout deck have a railing, while the fourth side is open. Determine the maximum area if there is 648 cm of railing.

Step1



$4w = 648$
$\div 4 \quad \div 4$
$w = 162$ cm
$= 1.62$ m

Step2

$2 \times 1.62 = 3.24$ m

$A = 1.62 \times 3.24$
$= 5.2488$

$\therefore$ It's approximately $5.25 m^2$

5. The area of a rectangular box is 722 500 mm^2. Determine the dimensions that minimize the perimeter.

$w^2 = 722500$
$\sqrt{w^2} = \sqrt{722500}$
$w = 850$ mm

$\therefore$ 85 cm x 85 cm

COMPLETE: p. 94 #1b,2b, 3b, 12,13,16