ACTIVITY *– Exploring* $a^{\frac{1}{n}}$

Examine the entries in the tables below. Determine the pattern to complete the next entries in each table.

Exponent 2	**Exponent –2**	**Exponent $\frac{1}{2}$**
$1^2 = 1$	$1^{-2} = 1$	$1^{\frac{1}{2}} = 1$
$2^2 = 4$	$2^{-2} = \frac{1}{4}$	$4^{\frac{1}{2}} = 2$
$3^2 = 9$	$3^{-2} = \frac{1}{9}$	$9^{\frac{1}{2}} = 3$
$4^2 = 16$	$4^{-2} = \frac{1}{16}$	$16^{\frac{1}{2}} = 4$
$5^2 = 25$	$5^{-2} = \frac{1}{25}$	$25^{1/2} = 5$
$6^2 = 36$	$6^{-2} = \frac{1}{36}$	$36^{1/2} = 6$
$7^2 = 49$	$7^{-2} = \frac{1}{49}$	$49^{1/2} = 7$

Exponent 3	**Exponent –3**	**Exponent $\frac{1}{3}$**
$1^3 = 1$	$1^{-3} = 1$	$1^{\frac{1}{3}} = 1$
$2^3 = 8$	$2^{-3} = \frac{1}{8}$	$8^{\frac{1}{3}} = 2$
$3^3 = 27$	$3^{-3} = \frac{1}{27}$	$27^{\frac{1}{3}} = 3$
$4^3 = 64$	$4^{-3} = \frac{1}{64}$	$64^{\frac{1}{3}} = 4$
$5^3 = 125$	$5^{-3} = 1/125$	$125^{1/3} = 5$
$6^3 = 216$	$6^{-3} = 1/216$	$216^{1/3} = 6$
$7^3 = 343$	$7^{-3} = 1/343$	$343^{1/3} = 7$

Compare the entries in the first and second column of each table. Describe the relationship that you see.

When the exponent is negative, the negative sign reciprocates (flipps) the base. The whole number base becomes a rational (fraction) number.

Compare the entries in the first and third column. What do you think it means to raise a number to an exponent of ½ or ⅓?

If the exponent is 1/2, it appears to be square rooting the base. For example 25 to the exponent 1/2 is 5. I know when I square root 25, the answer is 5.
If the eponent is 1/3, it also appears to be cube rooting the base. For example 8 to the exponent 1/3 is 2. I know that when I cube root 8, the answer is 2.

Use your results above to define a formula for $a^{\frac{1}{n}} = \sqrt[n]{a}$

e.g. $25^{\frac{1}{2}} = \sqrt[2]{25} = \sqrt{25} = 5$ (we omit the 2)

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

ACTIVITY **– *Exploring*** $a^{\frac{m}{n}}$

Examine the entries in the tables below. Use your calculator to complete each table.

To do a fractional (rational) exponent on your calculator you will need to:

- Use exponent button on your calculator (either the x^y, y^x, or ^ button)
- Use brackets around the fraction
- For example: Enter $25^{\frac{3}{2}}$ as **25** x^y **(3 ÷ 2) =**

square root

a	$a^{\frac{1}{2}}$	$a^{\frac{3}{2}}$	$a^{\frac{5}{2}}$
1	1	$1^{3/2}=1$	$1^{5/2}=1$
4	2	$4^{3/2}=8$	$4^{5/2}=32$
9	3	$9^{3/2}=27$	$9^{5/2}=243$
16	4	$16^{3/2}=64$	$16^{5/2}=1024$

cube root

a	$a^{\frac{1}{3}}$	$a^{\frac{2}{3}}$	$a^{\frac{5}{3}}$
1	1	1	1
8	2	4	32
27	3	9	243
64	4	16	1024

Compare the entries in the second, third, and fourth columns of each table.

How do the values of $a^{\frac{3}{2}}$ and $a^{\frac{5}{2}}$ relate to the values of $a^{\frac{1}{2}}$?

if you take 4 as a $4^{1/2}=2$ $4^{3/2}=8$ $4^{5/2}=32$

cubed $=2^3$

to the exponent 5 2^5

How do the values of $a^{\frac{2}{3}}$ and $a^{\frac{5}{3}}$ relate to the values of $a^{\frac{1}{3}}$?

Same as above. $a^{1/3}$ cube root $a^{2/3}$ → cube root then square $a^{5/3}$ → cube root then raise to exponent 5

Use your results above to define a formula for $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$

e.g. $8^{\frac{2}{3}}$ $\left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$