

## PERCENTILE and QUARTILE

Recall Measures of Central Tendency ( mean, median, and mode )

Mean: the average (add up all the values and divide by the # of values in the data set)

Median: the value that lies in the **middle** of **sorted** data

Mode: the value that occurs most frequently within the data

Range: the highest data value **MINUS** the lowest data value (a measure of spread)

### KEY WORDS

- Mean
- Median
- Mode
- Mean
- Median
- Mode
- Range

**EXAMPLE 1** The 14 students in Jesse's math class measured their heights to the nearest centimetre.

~~160~~ ~~178~~ ~~167~~ ~~180~~ ~~168~~ ~~157~~ ~~164~~ ~~179~~ ~~153~~ ~~182~~ ~~176~~ ~~165~~ ~~175~~ ~~167~~  
 1     2     3     4     5     6     7     8     9     10    11    12    13    14

a) Determine the measures of central tendency and the range for this set of data.

<b>MEAN</b>	* (add all numbers) / how many $\frac{2371}{14} = 169.4 \text{ cm}$ <p style="text-align: center; margin-left: 100px;">lowest to greatest ↑</p>
<b>MEDIAN</b>	* Rearrange the numbers in ascending order. If the set has even number of data, average the two numbers in the middle to find the median. If the set has odd number of data, then the median is the number in the middle. $\underbrace{153 \quad 157 \quad 160 \quad 164 \quad 165 \quad 167}_{5} \quad \boxed{167 \quad 168} \quad \underbrace{175 \quad 176 \quad 178 \quad 179 \quad 180 \quad 182}_{5}$ <p style="text-align: center; margin-left: 100px;">Average is 167.5cm</p>
<b>MODE</b>	* If one number repeated, mode 1; if two numbers repeated bimodal; if more than 3 numbers repeated multimodal. Repeated numbers 167; thus it is mode 1.
<b>RANGE</b>	* max - min = $182 - 153 = \underline{\underline{29}}$

b) What percent of the class is shorter than each measure of central tendency?

SHORTER THAN MEAN	SHORTER THAN MEDIAN	SHORTER THAN MODE
8 students are shorter than 169.4cm $\frac{8}{14} \times 100 = 57.14\%$	7 students are shorter than 167.5cm. $\frac{7}{14} \times 100 = 50\%$	Mode is 167. 5 students are shorter than 167cm. $\frac{5}{14} \times 100 = 35.71\%$

c) Ryan is taller than 65% of the class. How many students are shorter than he is? What is Ryan's height?

Number of students shorter than Ryan

$14 \times 65\% = 9$  shorter than Ryan Ryan

Ryan is 176cm tall.

$153 \quad 157 \quad 160 \quad 164 \quad 165 \quad 167 \quad 167 \quad 168 \quad 175 \quad \boxed{176} \quad 178 \quad 179 \quad 180 \quad 182$   
 1     2     3     4     5     6     7     8     9     10<sup>th</sup>

**MEASURES OF SPREAD**

**STANDARD DEVIATION:** measures how closely the data are centered around the mean

**PERCENTILES:** tells what percent of the data are less than a particular data value

- 20% of the data are less than or equal to the 20<sup>th</sup> percentile

**Example:** Bob is in the 23<sup>rd</sup> percentile for intelligence. This means:

- 23% of the population is less intelligent than Bob.
- 77% of the population is more intelligent than Bob.

**KEY WORDS**

Closely  
Mean  
Less than  
20%, 20<sup>th</sup>

TRY:

1	2	3	4	5	6	7	8	9	10	11	12
28	33	37	40	45	46	48	49	49	53	59	63

a) What percentile is 59?

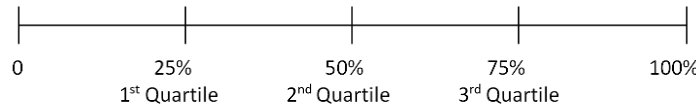
$\frac{10}{12} \times 100 = 83.3\%$       $\therefore$  83.3<sup>th</sup> percentile

b) Anyone in the 25<sup>th</sup> percentile is considered "young". What age is that?

25% of 12 = 3      $\therefore$  Age 40

**QUARTILES:** divide a set of sorted data into **four** equal parts

- The 2<sup>nd</sup> quartile,  $Q_2$  is the median of the entire data. It cuts the data set in half so it is the same as the 50<sup>th</sup> percentile.
- The 1<sup>st</sup> quartile,  $Q_1$  is the median of the lower half below  $Q_2$ . It divides the lower half of the data set in half so it is the same as the 25<sup>th</sup> percentile
- The 3<sup>rd</sup> quartile,  $Q_3$  is the median of above  $Q_2$ . It divides the upper half of the data set in half so it is the same as the 75<sup>th</sup> percentile



**EXAMPLE 1:** Find the quartiles of the data: ~~3, 7, 8, 5, 12, 14, 21, 13, 18~~ ← Check: There's 9 in each set

**Step 1:** Sort the numbers in ascending – lowest to greatest – order. (Ensure you fit all of them in one line).

**Step 2:** Circle the median (middle number). This is your 2<sup>nd</sup> quartile.

**Step 3:** Find the median of the lower half (set to the left of the median). This is the 1<sup>st</sup> quartile

**Step 4:** Find the median of the upper half (set to the right of the median). This is the 3<sup>rd</sup> quartile.

Step 1

3   5   7   8   12   13   14   18   21

Lower half     2<sup>nd</sup> quartile     Upper half

← 3   5   7   8     13   14   18   21

1<sup>st</sup> quartile     3<sup>rd</sup> quartile

$= \frac{5+7}{2} = 6$       $= \frac{14+18}{2} = 16$

$\therefore Q_1 = 6$   
 $Q_2 = 12$   
 $Q_3 = 16$

even quart we have to average 5 & 7

**EXAMPLE 2:** Find the quartiles of the data: 3, 5, 7, 8, 12, 14, 14, 15, 17, 21

Check: There are 10 in each set

3   5   7   8   12	14   14   15   17   21
Lower half	Upper half
3   5   7   8   12	14   14   15   17   21
1 <sup>st</sup> quartile = 7	3 <sup>rd</sup> quartile = 15

$Q_1 = 7$   
 $Q_2 = 13$   
 $Q_3 = 15$

**EXAMPLE 3:** The following dollar amounts were the hourly collections from a Salvation Army kettle at a local store one day in December: \$19, \$22, \$23, \$25, \$26, \$28, \$29, \$31, \$32, \$34, \$37, and \$39. Determine the first quartile and third quartile for the amount collected.

Check: 12 in each set

19   22   23   25   26   28	29   31   32   34   37   39
Lower half	Upper half
19   22   23   25   26   28	29   31   32   34   37   39
$Q_1 = \frac{23+25}{2} = 24$	$Q_3 = \frac{32+34}{2} = 33$

$Q_2 = \frac{28+29}{2} = 28.5$   
 $\therefore Q_1 = 24$   
 $Q_3 = 33$

**EXAMPLE 4:** Here are the hourly pay rates, in dollars, for 17 high-school students with part-time jobs.

- |       |       |       |       |      |      |      |      |      |
|-------|-------|-------|-------|------|------|------|------|------|
| 11.50 | 10.50 | 8.00  | 8.25  | 9.00 | 9.15 | 9.75 | 7.50 | 8.00 |
| 12.50 | 13.00 | 11.25 | 10.75 | 9.50 | 9.25 | 9.45 | 7.75 |      |

a) What are the quartiles for this data set?

7.50   7.75   8   8.25   9   9.15   9.25	(9.45)	9.50   9.75   10.50   10.75   11.25   11.50   12.50   13
Lower half	Q2	Upper half
middle #s		middle #s
$Q_1 = \frac{8+8.25}{2} = \$8.125$		$Q_3 = \frac{10.75+11.25}{2} = \$11$

$Q_1 = \$8.125$   
 $Q_2 = \$9.45$   
 $Q_3 = \$11$

b) Damien's pay is in the 85<sup>th</sup> percentile for this group. What does the percentile mean? What is Damien's hourly pay rate?

a) Damien is making more than 85% of 17 students.

$17 \times 85 \div 100 = 14$

b) Damien's pay is therefore 15<sup>th</sup> on the sorted list \$11.50.