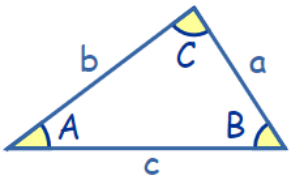
THE SINE LAW

So far, we've used trigonometric ratios to solve right angle triangles. The sine rule can be used in any triangle (not just right-angled triangles) where a side and its opposite angle are known.



a, **b** and **c** are sides.
A, **B** and **C** are angles.
(Side a faces angle A, side b faces angle B and side c faces angle C).

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$ (for finding sides)

or

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$ (for finding angles)

KEY WORDS
Sine rule
Side
Opposite
Length

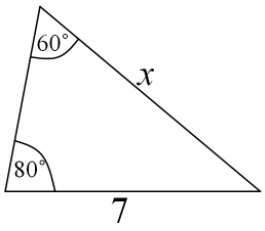
CASE 1) FINDING SIDE
GIVEN → ANGLE – ANGLE – SIDE

If you need to find the length of a side, you need to use the version of the Sine Rule where the lengths are on the top:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

You will only ever need two parts of the Sine Rule formula, not all three.
You will need to know at least one pair of a side with its opposite angle to use the Sine Rule.

Solved Example: Determine the length of x:



Step 1 Start by writing out the Sine Rule formula for finding sides:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

Step 2 Fill in the values you know, and the unknown length:

$$\frac{x}{\sin(80°)} = \frac{7}{\sin(60°)}$$

Remember that each fraction in the Sine Rule formula should contain a side and its opposite angle.

Step 3 Solve the resulting equation to find the unknown side, giving your answer to 3 significant figures:

$$\frac{x}{\sin(80°)} = \frac{7}{\sin(60°)}$$ *(multiply by sin(80°) on both sides)*

$$x = \frac{7}{\sin(60°)} \times \sin(80°)$$

$$x = 7.96$$

CASE 2) FINDING ANGLE
GIVEN → SIDE – SIDE – OPPOSITE ANGLE

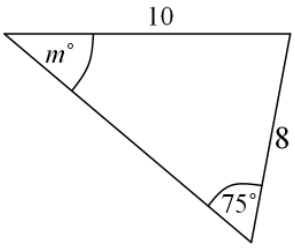
If you need to find the size of an angle, you need to use the version of the Sine Rule where the angles are on the top:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

As before, you will only need two parts of the Sine Rule , and you still need at least a side and its opposite angle.

Solved Example:

Determine the angle to the nearest degree:



Step 1 Start by writing out the Sine Rule formula for finding angles:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

Step 2 Fill in the values you know, and the unknown angle:

$$\frac{\sin(m°)}{8} = \frac{\sin(75°)}{10}$$

Remember that each fraction in the Sine Rule formula should contain a side and its opposite angle.

Step 3 Solve the resulting equation to find the sine of the unknown angle:

$$\frac{\sin(m°)}{8} = \frac{\sin(75°)}{10}$$ *(multiply by 8 on both sides)*

$$\sin(m°) = \frac{\sin(75°)}{10} \times 8$$

$$\sin(m°) = 0.773 \text{ (3 significant figures)}$$

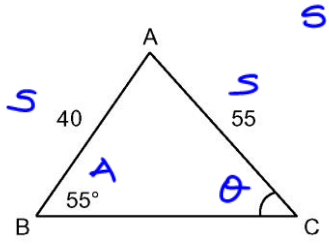
Step 4 Use the inverse-sine function ($\sin^{-1}$) to find the angle:

$$m° = \sin^{-1}(0.773) = 50.6° \text{ (3sf)}$$

Therefore, the angle is 51°.

PRACITCE:

1) Find the measure of $\angle C$ to the nearest tenth of a degree.



SSA

$\frac{\sin\theta}{40} \times 40 = \frac{\sin 55}{55} \times 40$

$\sin\theta = \frac{\sin 55 \times 40}{55}$

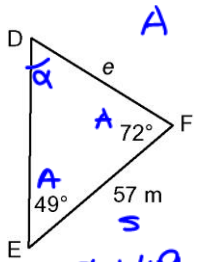
$\sin^{-1}\left(\frac{\sin 55 \times 40}{55}\right) = \theta$

$\theta \doteq 36.6°$

∴ θ is $36.6°$

SOLVE
To determine all unknown angles and sides.

2) Find the measure of side *e* to the nearest tenth.



$\frac{e}{\sin 49} = \frac{57}{\sin\alpha} \Rightarrow \frac{e}{\sin 49} = \frac{57}{\sin 59}$

$e = \frac{57}{\sin 59} \times \sin 49$

$e \doteq 50.2\text{ m}$

∴ e is 50.2 m

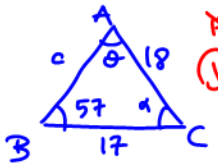
$\alpha + 49 + 57 = 180$

$\alpha = 180 - 49 - 72$

$\alpha = 59$

3) Solve the triangle *ABC* given $\angle B = 57°$, $a = 17$ cm, $b = 18$ cm

Steps
① Find θ using sine law
② use sum of angles in a triangle to solve α
③ find c using sine law



① $\frac{\sin\theta}{17} = \frac{\sin 57}{18} \times 17$

$\sin\theta = \frac{\sin 57 \times 17}{18}$

$\sin^{-1}\left(\frac{\sin 57 \times 17}{18}\right) = \theta$

$\theta \doteq 52.4°$

② $52.4 + 57 + \alpha = 180$

$\alpha = 180 - 52.4 - 57$

$\alpha = 70.6°$

③ $\frac{c}{\sin 70.6} = \frac{18}{\sin 57}$

$c = \frac{18 \times \sin 70.6}{\sin 57}$

$c \doteq 20.2\text{ cm}$

∴ ∠A(θ) is 52.4°
∠C(α) is 70.6°
c is 20.2cm

4) Solve for the unknown value to the nearest tenth

a) $\frac{a}{\sin 42°} = \frac{52}{\sin 68°} \times \sin 42$

$a = \frac{52 \times \sin 42}{\sin 68}$

$a \doteq 37.5$

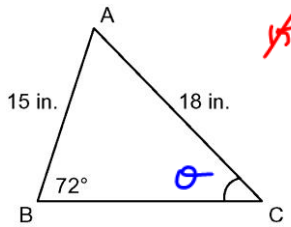
b) $\frac{\sin B}{11} = \frac{\sin 84°}{19} \times 11$

$\sin B = \frac{\sin 84 \times 11}{19}$

$\sin^{-1}\left(\frac{\sin 84 \times 11}{19}\right) = B$

$B \doteq 35.2°$

5) Find the measure of $\angle C$ to the nearest degree



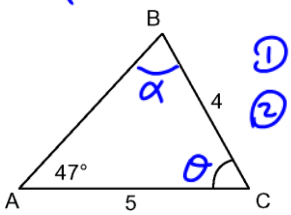
$\frac{\sin\theta}{15} = \frac{\sin 72}{18} \times 15$

$\sin\theta = \frac{\sin 72 \times 15}{18}$

$\sin^{-1}\left(\frac{\sin 72 \times 15}{18}\right) = \theta$

$\theta \doteq 52°$

∴ ∠C(θ) is 52°



① Find α first
② then using sum of interior angles find θ

① $\frac{\sin\alpha}{5} = \frac{\sin 47}{4}$

$\sin\alpha = \frac{\sin 47 \times 5}{4}$

$\sin^{-1}\left(\frac{\sin 47 \times 5}{4}\right) = \alpha$

$\alpha \doteq 66°$

② $\theta + 66 + 47 = 180$

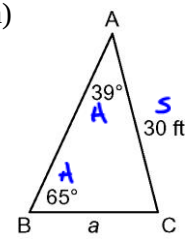
$\theta = 180 - 66 - 47$

$\theta = 67°$

∴ ∠C(θ) is 67°

6) Find the measure of the indicated side to the nearest tenth.

a)



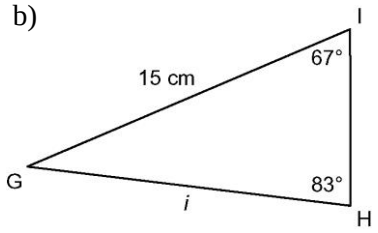
$\frac{a}{\sin 39} = \frac{30}{\sin 65} \times \sin 39$

$a = \frac{30 \times \sin 39}{\sin 65}$

$a \doteq 20.8$ ft

∴ side a is 20.8 ft

b)



$\frac{i}{\sin 67} = \frac{15}{\sin 83} \times \sin 67$

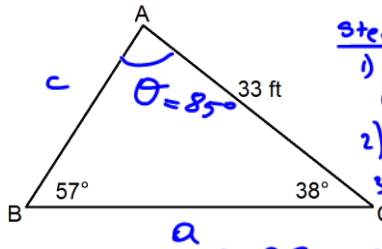
$i = \frac{15 \sin 67}{\sin 83}$

$i \doteq 13.9$ cm

∴ side i is 13.9 cm.

7) Solve each triangle *ABC*.

a)



Steps
1) solve θ using interior angles
2) solve for a
3) solve for c

Step1: $\theta = 180 - 57 - 38$

$\theta = 85°$

Step2: $\frac{a}{\sin 85} = \frac{33}{\sin 57}$

$a = \frac{33 \times \sin 85}{\sin 57}$

$a \doteq 39.2$ ft

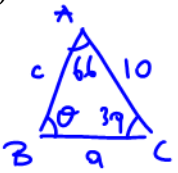
Step3: $\frac{c}{\sin 38} = \frac{33}{\sin 57}$

$c = \frac{33 \times \sin 38}{\sin 57}$

$c \doteq 24.2$ ft

∴ ∠A(θ) is 85°
side a is 39.2 ft
side c is 24.2 ft

b) Given $A = 66°$, $C = 39°$, $b = 10$



Steps
1) solve for θ
2) solve for a
3) solve for c

Step1: $\theta = 180 - 66 - 39$

$\theta = 75°$

Step2: $\frac{a}{\sin 66} = \frac{10}{\sin 75}$

$a = \frac{10 \times \sin 66}{\sin 75}$

$a \doteq 9.5$

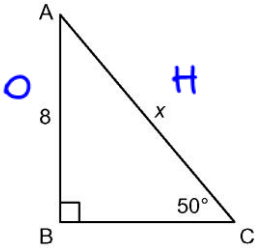
Step3: $\frac{c}{\sin 39} = \frac{10}{\sin 75}$

$c = \frac{10 \sin 39}{\sin 75}$

$c \doteq 6.5$

∴ ∠B is 75°
side a is 9.5
side c is 6.5

8) a) Use the **sine ratio** to find the value of *x*, to the nearest tenth



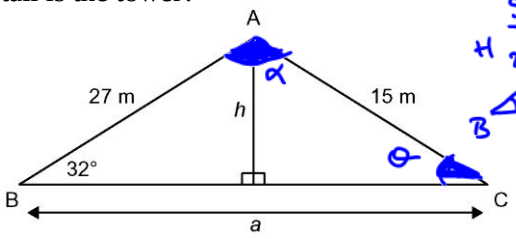
$\sin 50 = \frac{Opp}{Hyp} \Rightarrow \sin 50 = \frac{8}{x} \Rightarrow x = \frac{8}{\sin 50}$

$x \doteq 10.4$

b) Use the **sine law** to find the value of *x*, to the nearest tenth

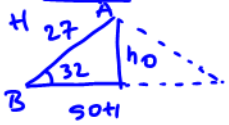
$\frac{x}{\sin 90} = \frac{8}{\sin 50} \Rightarrow x = \frac{8 \sin 90}{\sin 50} \Rightarrow x \doteq 10.4$

9) Two guy wires 27 m and 15 m in length are to be fastened to the top of a TV tower from two points B and C as shown. The angle of elevation to the top of the tower of the longer wire is $32°$. How far apart are points B and C and how tall is the tower?



Steps
1. solve for h using trig ratios
2. find ∠C using sine law
3. find ∠A (α)
4. find a using sine law

Step 1



$\sin 32 = \frac{h}{27}$

$h = 27 \sin 32$

$h = 14.3 m$

∴ h is 14.3 m
a is 27.4 m

Step 2

$\frac{\sin \theta}{27} = \frac{\sin 32}{15}$

$\sin \theta = \frac{\sin 32 \times 27}{15}$

$\sin^{-1}\left(\frac{\sin 32 \times 27}{15}\right) = \theta$

$\theta \doteq 72.5°$

Step 3

$\alpha = 180 - 72.5 - 32$

$\alpha = 75.5$

Step 4

$\frac{a}{\sin 75.5} = \frac{15}{\sin 32}$

$a = \frac{15 \sin 75.5}{\sin 32}$

$a \doteq 27.4$