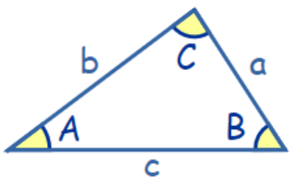
THE SINE LAW

So far, we have used trigonometric ratios to solve right angle triangles. The sine rule can be used in any triangle (not just right-angled triangles) where a side and its opposite angle are known.



a, **b** and **c** are sides.
A, **B** and **C** are angles.
(Side a faces angle A, side b faces angle B and side c faces angle C).

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$ (for finding sides)

or

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$ (for finding angles)

KEY WORDS
Sine rule
Side
Opposite
Length

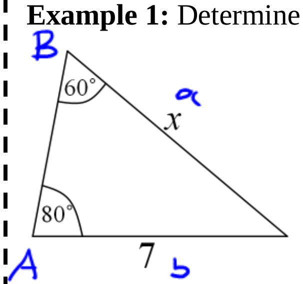
CASE 1) FINDING SIDE: ANGLE – ANGLE – SIDE

If you need to find the length of a side, you need to use the version of the Sine Rule where the lengths are on the top:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

You will only ever need two parts of the Sine Rule formula, not all three.
You will need to know at least one pair of a side with its opposite angle to use the Sine Rule.

Example 1: Determine the length of x:



Step1 : Write the formula

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{x}{\sin 80} = \frac{7}{\sin 60}$$

$x \div \sin 80 = 7 \div \sin 60$

Step2 : To cancel out the division by sin 80 on left side, multiply both sides by sin 80.

$x \div \sin 80 \times \sin 80 = 7 \div \sin 60 \times \sin 80$

$x \doteq 8$ approximately

∴ Side x is approximately 8 units.

CASE 2) FINDING ANGLE: SIDE – SIDE – OPPOSITE ANGLE

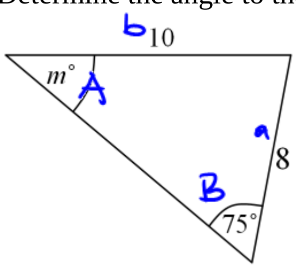
If you need to find the size of an angle, you need to use the version of the Sine Rule where the angles are on the top:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

As before, you will only need two parts of the Sine Rule , and you still need at least a side and its opposite angle.

Example 2:

Determine the angle to the nearest degree:



Step1: Set up the equation

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{8} = \frac{\sin 75}{10}$$

$$\sin A \div 8 = \sin 75 \div 10$$

Step2: isolate sin A by multiplying both sides by 8.

$$\sin A \div 8 \times 8 = \sin 75 \div 10 \times 8$$

$$\sin A = 0.7727$$

$$\sin^{-1}(0.7727) = A$$

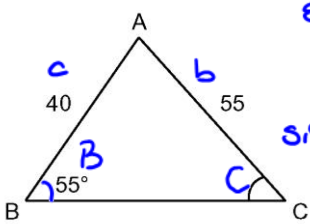
$$\boxed{A \cong 51°}$$

∴ Angle m is approximately 51°.

PRACTICE:

SOLVE
To determine all unknown angles and sides.

1) Find the measure of $\angle C$ to the nearest tenth of a degree.



$\sin C \div 40 \cdot 40 = \sin 55 \div 55 \cdot 40$

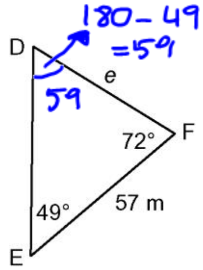
$\sin C = 0.5957$

$\sin^{-1}(0.5957) = C$

$\boxed{C \cong 37°}$

2) Find the measure of side *e* to the nearest tenth.

180 − 49 − 72
= 59

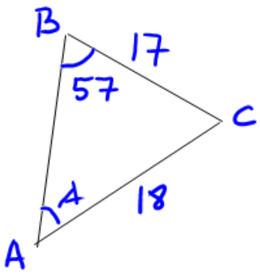


$e \div \sin 49 = 57 \div \sin 59$

$e \div \sin 49 \times \sin 49 = 57 \div \sin 59 \times \sin 49$

$\boxed{e \doteq 50}$

3) Solve the triangle *ABC* given $\angle B = 57°$, $a = 17$ cm, $b = 18$ cm



Step1: Find measure of ∠A.

$\sin A \div 17 = \sin 57 \div 18$

$\sin A \div 17 \times 17 = \sin 57 \div 18 \times 17$

$\sin A = 0.7921$

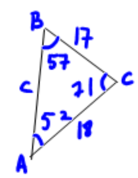
$\sin^{-1}(0.7921) = A$

$\boxed{A \cong 52°}$

Step2: Find measure of side c

$\angle C = 180 - 57 - 52$

$\angle C = 71°$



$c \div \sin 71 = 18 \div \sin 57$

$c \div \sin 71 \times \sin 71 = 18 \div \sin 57 \times \sin 71$

$\boxed{c \doteq 20}$

$\therefore \angle A = 52°$, $\angle B = 57$, $\angle C = 71$
$a = 17$, $b = 18$, $c = 20$

4) Solve for the unknown value to the nearest tenth

a) $\sin 42 \times \dfrac{a}{\sin 42°} = \dfrac{52}{\sin 68°} \times \sin 42$

$\boxed{a \doteq 37.5}$

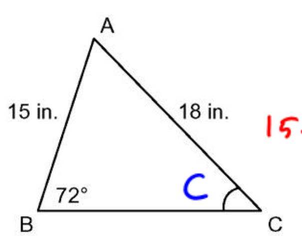
b) $11 \cdot \dfrac{\sin B}{11} = \dfrac{\sin 84°}{19} \cdot 11$

$\sin B = 0.5758$

$\sin^{-1}(0.5758) = B$

$\boxed{B \doteq 35.2}$

5) Find the measure of $\angle C$ to the nearest degree



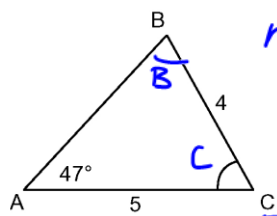
$\dfrac{\sin C}{15} = \dfrac{\sin 72}{18}$

$15 \cdot \dfrac{\sin C}{15} = \dfrac{\sin 72}{18} \cdot 15$

$\sin C = 0.7925$

$\sin^{-1}(0.7925) = C$

$\boxed{C \doteq 52°}$



need to find ∠B first

$\dfrac{\sin B}{5} = \dfrac{\sin 47}{4}$

$5 \cdot \dfrac{\sin B}{5} = \dfrac{\sin 47}{4} \cdot 5$

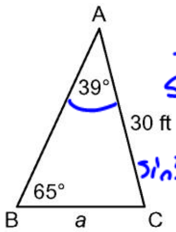
$\sin B = 0.9142$

$\sin^{-1}(0.9142) = B$

$\boxed{B \doteq 66°}$

6) Find the measure of the indicated side to the nearest tenth.

a)

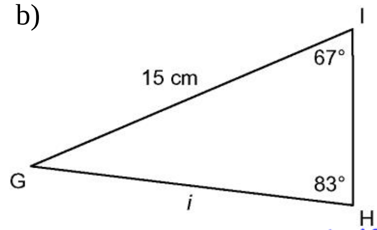


$\frac{a}{\sin 39} = \frac{30}{\sin 65}$

$\sin 39 \cdot \frac{a}{\sin 39} = \frac{30}{\sin 65} \cdot \sin 39$

$\boxed{a \doteq 20.8}$

b)



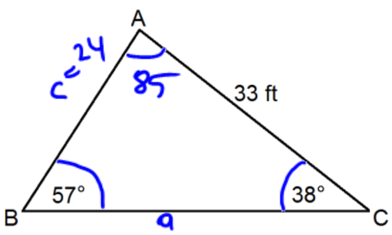
$\frac{i}{\sin 67} = \frac{15}{\sin 83}$

$\sin 67 \cdot \frac{i}{\sin 67} = \frac{15}{\sin 83} \cdot \sin 67$

$\boxed{i \doteq 13.9}$

7) Solve each triangle *ABC*.

a)



Step 1: side c

$c \div \sin 38 = 33 \div \sin 57$

$c \div \sin 38 \times \sin 39 = 33 \div \sin 57 \times \sin 38$

$\boxed{c = 24}$

Step 2: side a $\angle A = 180 - 57 - 38$

$\angle A = 85°$

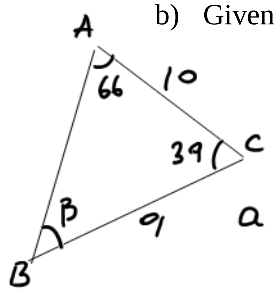
$a \div \sin 85 \times \sin 85 = 33 \div \sin 57 \times \sin 85$

$\boxed{a = 39}$

$\therefore \angle A = 85;\ \angle B = 57;\ \angle C = 38$

$a = 39\text{ ft}\quad b = 33\text{ ft},\quad c = 24$

b) Given $A = 66°$, $C = 39°$, $b = 10$



$\angle B = 180 - 66 - 39$

$\angle B = 75°$

Step 1: Find side a

$a \div \sin 66 = 10 \div \sin 75$

$a = 10 \div \sin 75 \times \sin 66$

$\boxed{a \doteq 9}$

Step 2: Find side c

$c \div \sin 39 = 10 \div \sin 75$

$c = 10 \div \sin 75 \times \sin 39$

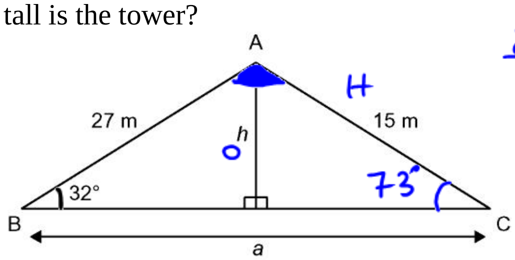
$\boxed{c \doteq 7}$

$\therefore \angle A = 66 \quad a = 9$

$\angle B = 75 \quad b = 10$

$\angle C = 39 \quad c = 7$

8) Two guy wires 27 m and 15 m in length are to be fastened to the top of a TV tower from two points B and C as shown. The angle of elevation to the top of the tower of the longer wire is $32°$. How far apart are points B and C and how tall is the tower?



Step 1: Find $\angle C$

$\sin C \div 27 = \sin 32 \div 15$

$\sin C = \sin 32 \div 15 \times 27$

$\sin C = 0.9539$

$\sin^{-1}(0.9539) = C$

$\boxed{C = 73°}$

Step 2: Find side a

$\angle A = 180 - 32 - 73$

$\angle A = 75$

$a \div \sin 75 = 15 \div \sin 32$

$a = 15 \div \sin 32 \times \sin 75$

$a = 27.34$

Step 3: $15 \cdot \sin 73 = \frac{h}{15} \cdot 15$

$15 \cdot \sin 73 = h$

$\boxed{h \doteq 14.34}$

$\therefore$ B and C are 27.34 m apart; the height of the tower is 14.34 m.