MAP4C **Date: ____________**
Day 5: Optimizing SA and Volume **Unit 2: Measurement & Geometry**

CASE 1) MINIMIZING SURFACE AREA: FIXED VOLUME, LOWEST SURFACE AREA

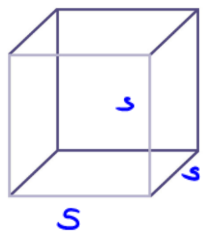
Problem: Guy has 64 m^3 of sand and wants to make a box to store it, using as little material as possible. Determine which box will have the **least surface area**?

*In the problem above, Guy is dealing with a situation where **volume** is fixed and he needs to create a box with the **lowest surface area** so that he will spend **the least** amount of money in material.*

The closer the box gets to being a _cube______, the smaller the **surface area** is for a given volume.

How to solve the problem above algebraically

*We know that **cube** will provide the lowest surface area for a fixed volume of 64 m^3 of sand. You need a calculator that has cube root button.*



Let "s" represent one side

$s^3 = 64$

$\sqrt[3]{s^3} = \sqrt[3]{64}$

$\boxed{s = 4}$

Tech Tip for 2 possible buttons

$\sqrt[x]{\ }$

1) Type 3 for the root
2) Press $\sqrt[x]{\ }$
3) Type 64

$\sqrt[3]{x}$

1) Type 64
2) Press $\sqrt[3]{x}$

∴ The cubic box which has the dimensions of 4m by 4m by 4m will have the least surface area.

TRY: State the dimensions that will **minimize** (lowest) the surface area of a shadow box that has a volume of 35937 cm^3.

$\sqrt[3]{s^3} = \sqrt[3]{35937}$

∴ 33cm by 33cm by 33cm

$s = 33$

CASE 2 – MAXIMIZING VOLUME: FIXED SURFACE AREA, MAXIMUM VOLUME

Dorsa has 24 m^2 of wood to make a toy box. Determine which box will have the maximum volume.

In the problem ~~below~~, Dorsa has a fixed amount of material (surface area) and needs to create a box that will provide the largest volume to fit the most amount of toys.

The closer the box gets to being a _cube______, the **larger** the volume is for a given surface area.

How to solve the problem above algebraically *We know that cube will provide the largest volume.*
Determine the dimensions of a box that maximizes the volume and has a surface area of 54 cm^2.

$6s^2 = 54$
$\div 6 \quad \div 6$

$s^2 = 9$

$\sqrt{s^2} = \sqrt{9}$

$\boxed{s = 3}$

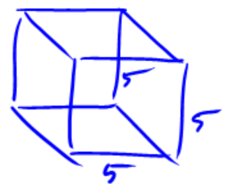
∴ length = 3 cm
width = 3 cm
height = 3 cm

Questions

1. A magician has ordered a covered water tank for his next new act. He has enough money to pay for 150 m^2 of building material. What is the largest volume of water that can be held in his water tank?

Cube will hold the largest volume of water

$6s^2 = 150$
$\div 6 \quad \div 6$
$s^2 = 25$
$\sqrt{s^2} = \sqrt{25}$
$\boxed{s = 5}$



$V = 5 \times 5 \times 5$

The largest volume of water is 125 m^3.

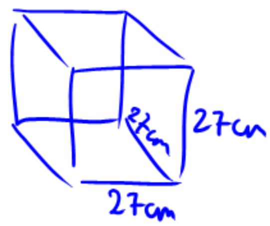
2. State the dimensions that will minimize the surface area of a shadow box that has a volume of 19683 cm^3.

$s^3 = 19683$ cube root both sides

$\sqrt[3]{s^3} = \sqrt[3]{19683}$

$s = 27$ cm

∴

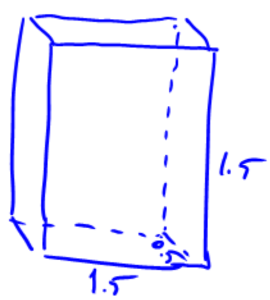


3. You have been asked to make a single shelf cabinet, with a volume of ~~4.5~~ 1.125 m^3. However, it can only be 0.5 m deep.

a) Determine the dimensions that will minimize the surface area.
b) Assuming that the front face of the shelf is open, what total surface area of wood is needed?

a) $s \cdot s \cdot 0.5 = 1.125$
$s^2 \times 0.5 = 1.125$
$\div 0.5 \quad \div 0.5$
$s^2 = 2.25$
$\sqrt{s^2} = \sqrt{2.25}$
$\boxed{s = 1.5}$

∴ width = 1.5 m
Height = 1.5 m
depth = 0.5 m



b) S.A = Back + Top + Bottom + LS + RS
$= 1.5 \times 1.5 + 1.5 \times 0.5 + 1.5 \times 0.5 + 1.5 \times 0.5 + 1.5 \times 0.5$
$= 5.25 \ m^2$

∴ The total surface area of wood needed is 5.25 m^2

COMPLTE: p. 110 #1, 2, 12, 18 and 19