Characteristics of Exponential Relations

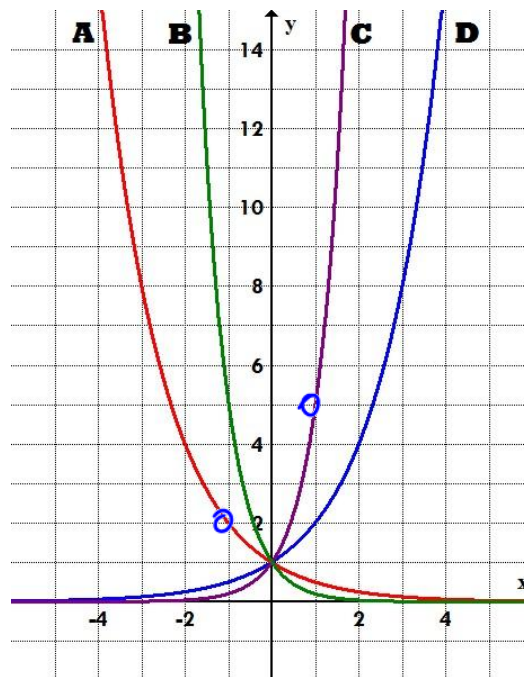Consider the following four equations and four graphs. Identify which graph represents each exponential relation.

____D____ $y = 2^x$ ____C____ $y = 5^x$

____A____ $y = \left(\dfrac{1}{2}\right)^x$ ____B____ $y = \left(\dfrac{1}{5}\right)^x$

What does each of the exponential relations have in common?

They all have the same y-int.

Which exponential relations show a GROWTH relationship?

C & D

Which exponential relations show a DECAY relationship?

A & B

CHARACTERISTICS OF THE EXPONENTIAL FUNCTION $y = a^x$

- The y-intercept is always __(0,1)__

- The graph of an exponential relation does not have an __x-int__

- The growth/decay factor is the __base__ of the power → "__a__" in $y = a^x$

EXPONENTIAL GROWTH

- If $a > 1$, the graph **increases** very slowly for __negative__ x-values and **increases** rapidly for __positive__ x-values. The graph is almost horizontal on the __left__ and approaches __vertical__ on the right.

- The bigger the value of a, the __faster__ the growth

EXPONENTIAL DECAY

- If $0 < a < 1$, the graph **decreases** very rapidly for __negative__ x-values and **decreases** slowly for __positive__ x-values. The graph is almost __vertical__ on the left and approaches horizontal on the __right__.

- The smaller the value of a, the __faster__ the decay

An Introduction to GROWTH and DECAY

GROWTH: York Region's population, P, is projected to grow until 2031 based on the relation $P = 610\ 000(1.029)^n$, where n is the number of years after 1996.

a) Sketch the graph of this relation.

# Years After 1996	Population
0	610,000
10	811864
20	1080,531
30	1438,106
40	1,914,012
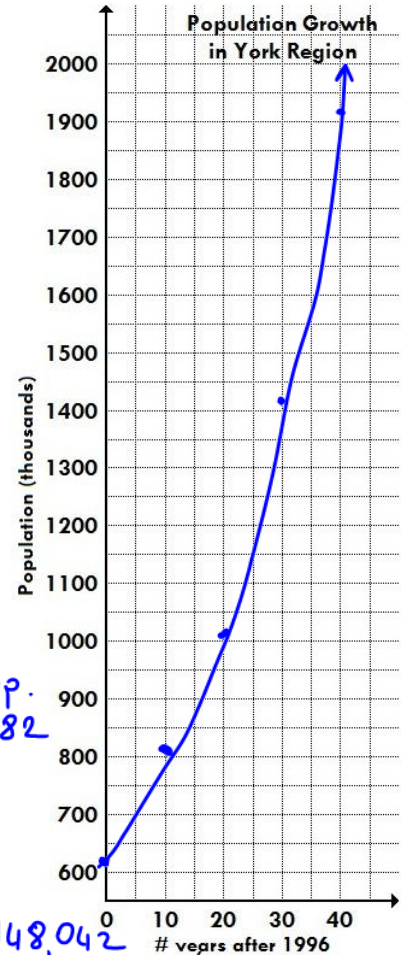
b) What is the P-intercept? What does it represent?

610,000 . Initial Population

c) What is the projected population in York Region in 2031?

2031 − 1996 = 35   n = 35 years

$P = 610000(1.029)^{35}$

≈ 1,659,082

∴ The project pop. in 2031 is 1,659,082

d) The actual population at the end of 2009 was 1 032 606. How far off from the projected population was the actual population?

2009 − 1996 = 13   n = 13 years.

$P = 610000(1.029)^{13}$

= 884,564

1,032,606 − 884,564 = 148,042

∴ Population grew faster than projected by 148,042

Graph: Population Growth in York Region
(Population (thousands) vs. # years after 1996)

DECAY: A pressure reader is used to measure the sound intensity of a bell. The relation $P = 200(0.5)^t$ estimates the sound pressure, P, in Pascals after t seconds. The graph of this relation is shown.

a) What is the P-intercept? What does it represent?

$= 200(0.5)^0$

$= 200$

∴ 200 is the initial intensity (sound pressure)

b) What is the sound pressure after:

i. 1 second

The pressure is 100 Pa at 1 sec.

ii. 2 seconds

The pressure has decreased to 50 Pa after 2 sec.

iii. 9 seconds

$P = 200(0.5)^9$

= 0.39

∴ After 9 sec. the sound pressure is only 0.39 Pa.

Graph: Sound Intensity of a Bell
(Pressure (Pa) vs. Time (s))