COMPOUND INTEREST

Simple Interest

- Interest paid on ONLY the principal of an investment or loan.
- Has a linear growth.

Compound Interest

- Interest paid on the principal **AND** its accumulated interest.
- Calculated at regular compounding periods and added to the principal for the next compounding period.
- Has an exponential growth.

KEY WORDS
- principal
- linear
- principal
- interest
- periods
- added
- exponential
- Accumulated amount
- principal
- interest rate in decimal
- compounding
- compounding

COMPOUND INTEREST FORMULA $A = P(1+i)^n$

A = Accumulated amount (or future value)

P = Principal (the initial amount)

i = interest rate in decimal per compounding period

n = number of compounding periods

Compounding Frequency Terminology

- Annually – once a year
- Semi-annually – 2 times per year (every 6 months)
- Quarterly – 4 times per year (every 3 months) $12 \div 3$
- Semi-monthly – 24 times per year (twice a month) 12×2
- Bi-weekly – 26 times per year (every 2 weeks) $52 \div 2 = 26$
- Weekly – 52 times per year (but **NOT** 4 times a month)

Interest Rate (i)

Calculate the interest rate (i) as it would appear in the compound interest formula.
(Hint: Convert to decimal and divide by the number of compounding periods)

a) 6% per year, compound semi-annually
$0.06 \div 2$
$= 0.03$ or 3%

b) 5% per year, compound weekly
$0.05 \div 52 = 0.00096$

c) 1.75% per year, compound quarterly
$0.0175 \div 4 = 0.004375$

Compounding Periods (n)

Calculate the number of compounding periods (n) as it would appear in the compound interest formula. *(Hint: multiply the length of time (in years) by the # of compounding periods in the compounding frequency)*

a) Compounded **quarterly** for 5 years

Year 1 = 4
Year 2 = 4
" 3 = 4
" 4 = 4
" 5 = 4

$n = 4 \times 5$
$= 20$ periods

b) Compounded **semi-annually** for 18 months

2 periods in 1 year
n in 1.5 years

$n = 2 \times 1.5$
$\boxed{n = 3}$

c) Compounded **bi-weekly** for 6 months

26 periods in 1 year
n in 0.5 years

$n = 26 \times 0.5$
$\boxed{n = 13}$

EXAMPLE 1

a) Calculate the amount of a \$500 investment, invested at 3% per year, compounded quarterly for 3 years.

Step 1
Type = Compound quarterly
$A = ?$
$P = \$500$
$i = 3\%/1\text{ year} = 0.03 \div 4 = 0.0075$
$n = 3\text{ years} = 3 \times 4 = 12$
has to match

Step 2
$A = P(1+i)^n$
$= 500(1+0.0075)^{12}$
$= \$546.90$
∴ Investment grows to \$546.90 in 3 years

b) How much interest was earned? I = A – P

$I = 546.90 - 500$
$= \$46.90$
∴ Total interest earned was \$46.90 in 3 years.

EXAMPLE 2

Peter borrowed \$5 000 to buy a used car. The interest rate on the loan was 5.45% per year, compounded monthly. He plans to repay the loan in four years.

a) How much must Peter repay?

Type = compound monthly
$A = ?$
$P = \$5000$
$i = 5.45\%/1\text{ year} \xrightarrow{\div 12} 0.0545 \div 12$ leave as for more accuracy
$n = 4\text{ years} \xrightarrow{\times 12} 4 \times 12 = 48$

$A = P(1+i)^n$
$= 5000(1+0.0545 \div 12)^{48}$
$= \$6214.87$
∴ He will pay \$6214.87 in 4 years

b) If Peter repays the loan 6 months early, how much interest will he save (not have to repay)?

n decreases by 6 = 42 periods
Step 1
$A = P(1+i)^n$
$= 5000(1+0.0545 \div 12)^{42}$
$= \$6048.17$

Step 2
The difference is $6214.87 - 6048.17$
\$166.7

EXAMPLE 3

Jennifer's investment has grown by an average of 12.6% per year, **compounded annually**, over the past seven years. How much would her investment of \$2000 be worth today?

Type : compounded annually
Step 1
$A : ?$
$P : \$2000$
$i : 12.6\%/1\text{ year}$
$n : 7\text{ years}$

Step 2
$A = P(1+i)^n$
$= 2000(1+0.126)^7$
$= 4589.85$
∴ It will be worth \$4589.85 in seven years.

COMPOUND INTEREST PRACTICE

1) Evaluate. Round answers to 2 decimal places.

a) $1000(1.0097)^{12}$
$= \$1122.82$

b) $575(1+0.0234)^{26}$
$= 1049.18$

c) $900(1+\frac{0.3}{12})^{24}$
$= 1627.85$

2) Calculate the interest rate (i) as it would appear in the compound interest formula.
(Hint: Convert to decimal and divide by the number of compounding periods)

d) 5% quarterly
$= 0.05 \div 4$
$= 0.0125$

e) 0.3% semi-annually
$= 0.003 \div 2$
$= 0.0015$

f) 1.25% monthly
$= \underbrace{0.0125 \div 12}$
leave it like that if you need to round

g) 4.2% bi-weekly
$= 0.042 \div 26$

h) 0.05% daily
$= 0.0005 \div 365$

i) 12% annually
$= 0.12$

3) Calculate the number of compounding periods (n) as it would appear in the compound interest formula. *(Hint: multiply the length of time (in years) by the # of compounding periods in the compounding frequency)*

a) Monthly for 2 years
$= 2 \times 12$
$= 24$

b) Weekly for 3 years
$= 3 \times 52$
$= 156$

c) Annually for 36 months
3 years
$= 3$

d) Semi-annually for 30 months
30 months $= \frac{30}{12} = 2.5$ years
$n = 2.5 \times 2$
$= 5$

e) Bi-weekly for 6 months
6 months $= \frac{6}{12} = 0.5$ years
$n = 0.5 \times 26$
$= 13$

f) Daily for 3 weeks
3 weeks $= \frac{3}{52}$
$n = \frac{3}{52} \times 365$

4) Jared needs to borrow \$3 000. Which loan should he take? Explain.
a) \$3 000 for five years at 9% per year, compounded semi-annually
b) \$3 000 for five years at 8.5% per year, compounded quarterly

<u>9% loan</u>	<u>8.5% loan</u>
Type = semi-annual	Type = quarterly
$A=$?	$A=$?
$P=$ 3000	$P=$ 3000
$i=$ 9%/1 year $= 0.09 \div 2 = 0.045$	$i=$ 8.5%/1 year $= 0.085 \div 4 = 0.02125$
$n=$ 5 years $\times 2 = 10$	$n=$ 5 years $\times 4 = 20$ $\boxed{60}$
$A = P(1+i)^n$	$A = P(1+i)^n$
$= 3000(1+0.045)^{10}$	$= 3000(1+0.02125)^{20}$
$= \$4658.90$	$= \$4568.58$

$\therefore$ He should take 8.5% loan because he'll pay \$90.52 less in interest.

5) The city of Melville has a population of 102 000 and a projected growth rate of 2.3% per year, for the next 10 years. The city of Markton has a population of 97 000 and a projected growth rate of 3.7% per year for the next 10 years. Which city is expected to have the greater population in 10 years?

Melville

Type = compound annually

A= 102000 (1 + 2.3%)^{10} = 128043

P= 102000

i= 2.3%/1 year

n= 10 years

Markton

Type : compound annually

A= 97000 (1 + 3.7%)^{10} = 139495

P= 97000

i= 3.7%/1 year

n= 10 years

Therefore…

Markton will have the greater population in 10 years.

6) The Stereo Warehouse is advertising "No money down and pay no interest for one year!" Peter read the fine print and discovered that, although you pay no interest for one year, interest is calculated at 12% per year, compounded monthly, on the price of the merchandise. What would Peter have to pay for an $1150 LCD TV after the one-year interest free period is over?

Type : compound monthly

A:

P: 1150

i: 12%/1 year = 0.12 ÷ 12 = 0.01

n: 1 year = 12 periods

$A = P(1+i)^n$

$= 1150(1 + 0.01)^{12}$

$= 1295.85$

∴ He will pay $1295.85 in total.

7) Mohammed spent $800 on his credit card. His credit card company charged 18% compounded monthly. He forgot to pay it for 3 months. How much does he owe now? How much of that is interest?

Type = compound monthly

A = ?

P = $800

i = 18%/1 year = 0.18 ÷ 12

n = 3 months = 3

$A = P(1+i)^n$

$= 800(1 + 0.18 \div 12)^3$

= $836.54

∴ He owes $836.54 in total and $36.54 is the interest charged for 3 months.

8) Congratulations, you just won $500 000 in the lottery. After buying a car, donating to your favourite charity and sharing some of your wealth with family and friends you decide to invest $200 000 for retirement. You put your money into a mutual fund which on average earns 6.5% per year, compounded annually. How much money will you have in 30 years?

Type : compound annually

A : ?

P : 200000

i : 6.5%/1 year

n : 30 years

$A = P(1+i)^n$

= 200000(1 + 6.5%)^{30}

= $1,322,873.23

∴ I will have $1,322,873.23