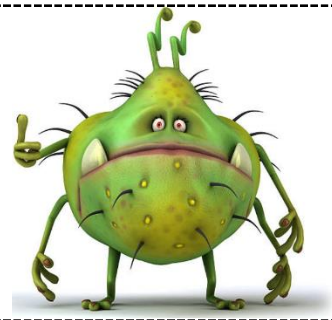
Germs! Germs! Germs!

Certain bacteria, under the right conditions, multiply themselves.



You will use strips of paper, each representing a bacterium, to model its growth.
For this activity, each member of your group must choose a role:

- Recorder – records data
- Counter – counts pieces for recorder
- Reader – reads questions for other members in the group
- Facilitator – keeps discussion of topic going

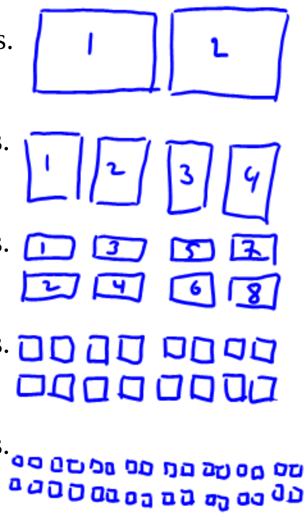
Cut #1: Cut your paper into 2 equal pieces.
How many total pieces do you have? 2

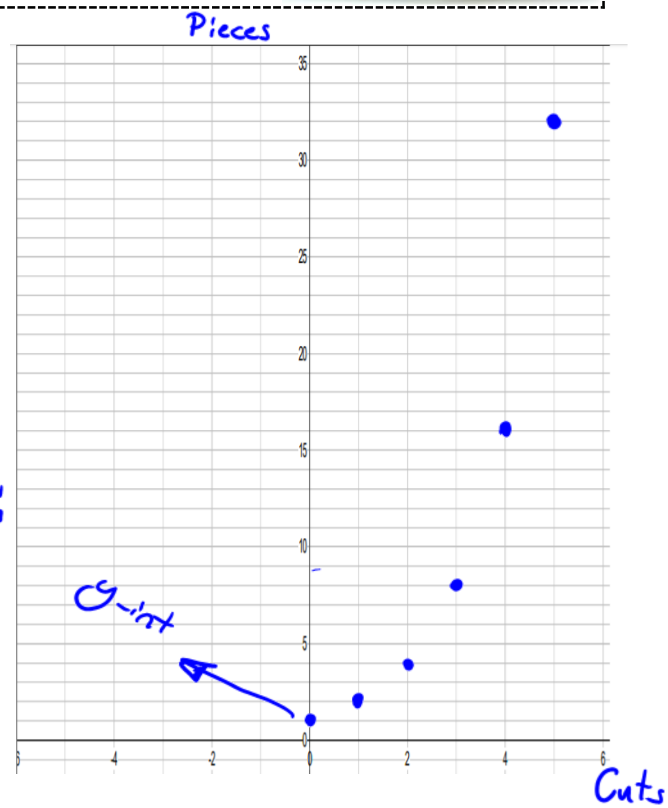
Cut #2: Cut each piece into 2 equal pieces.
How many total pieces do you have? 4

Cut #3: Cut each piece into 2 equal pieces.
How many total pieces do you have? 8

Cut #4: Cut each piece into 2 equal pieces.
How many total pieces do you have? 16

Cut #5: Cut each piece into 2 equal pieces.
How many total pieces do you have? 32





Initial

	Cuts	Pieces	Growth factor
initial	0	1	
	1	2	$2/1 = 2$
	2	4	$4/2 = 2$
	3	8	$8/4 = 2$
	4	16	$16/8 = 2$
	5	32	$32/16 = 2$

Graph your result on the grid provided.
Identify the characteristics of your graph.

- X – intercept NONE
- Y - intercept initial amount

Create an equation to model the data in $y = ab^x$ form.

$$y = 1 \cdot 2^x$$

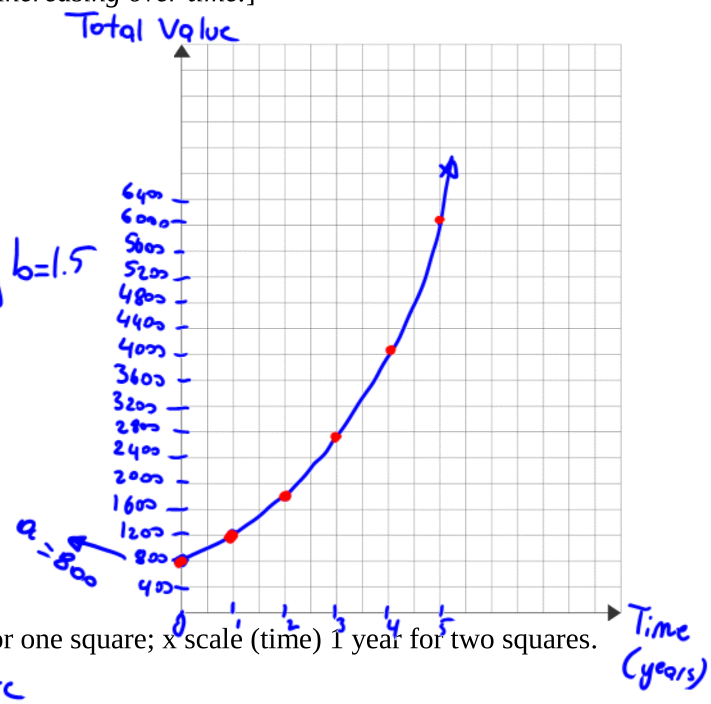
$$\therefore y = 2^x$$

y is total amount (leave y as is)
a is initial amount
b is growth factor (# of equal pieces each cut)
x is number of cuts (leave x as is)

EXAMPLE 2:
An antique costs $800. Its value increases at a rate of 50% each year. **Fill** out the chart below.
[***NOTE****: 50% is called the* ***growth rate*** *because the initial amount is increasing over time.*]

End of Year	Increase in Value	Total Value at the end of Year	Growth Factor (b)
0		= $800 * *initial value (a)*	
1	= 800 x 50% = 800 x 0.50 = $400	= 800 + 400 = $1200	1200 / 800 = 1.5
2	= 1200 x 0.50 = $600	= 1200 + 600 = $1800	1800/1200 = 1.5
3	= 1800 x 0.50 = $900	= 1800 + 900 = $2700	2700/1800 = 1.5
4	= 2700 x 0.50 = $1350	= 2700 + 1350 = $4050	4050/2700 = 1.5
5	= 4050 x 0.50 = $2025	= 4050 + 2025 = $6075	6075/4050 = 1.5

$b = 1.5$



Graph your result on the grid provided. Y scale (Total Value) 400 for one square; x scale (time) 1 year for two squares.
Identify the characteristics of your graph
x – Intercept: none
y – Intercept: $y = 800$ initial value
Equation:

growth rate

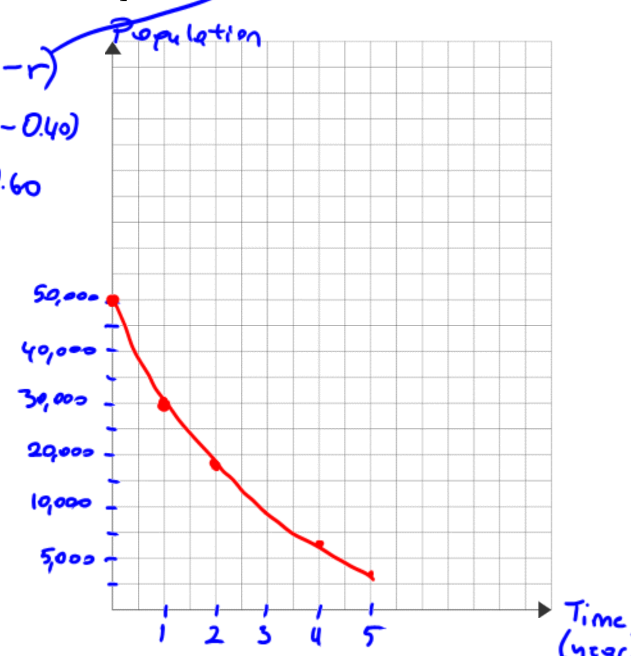
$y = 800 \cdot 1.5^x$ or $y = 800(1 + 0.50)^x$

decay rate -0.40

EXAMPLE 3: The population of Luckville is 50,000. Each year, the population decreases at a rate of 40%.
[***NOTE****: 40% is called the decay rate because initial amount is decreasing over time.*]

End of Year	Decrease in Population	Total Population at the end of Year	Decay Factor (b)
0		50,000 * *initial value* (a)	
1	= 50,000 x 40% = 50,000 x 0.40 = 20,000	= 50,000 – 20,000 = 30,000	30,000/50,000 = 0.60
2	= 30,000 x 0.40 = 12,000	= 30,000 − 12,000 = 18,000	18000/30000 = 0.60
3	= 18,000 x 0.40 = 7200	= 18000 − 7200 = 10,800	10800/18000 = 0.60
4	= 10800 x 0.40 = 4320	10800 − 4320 = 6480	6480/10800 = 0.60
5	= 6480 x 0.40 = 2592	= 6480 − 2592 = 3888	3888/6480 = 0.60

$(1 - r)$
$(1 - 0.40)$
0.60



Graph your result on the grid provided. Y scale (Total Value) 2500 for one square; x scale (time) 1 year for two squares.
Identify the characteristics of your graph
x – Intercept: none
y – Intercept: $a = 50,000$ initial
Equation:
$y = 50000(0.60)^x$

Decay Factor = 1 − Decay Rate
$b = 1 - r$

Exponential models represent quantities that change at a constant _percent rate_; that is, the quantity is _multiplied_ by a fixed amount at regular intervals.

- In a table of values, the _growth_ / _decay_ factors are equal
- The graph resembles an _exponential_ curve
- The equation is written in the form _$y = ab^x$_ where _a_ is the initial value and _b_ is the growth/decay factor. Notice that the exponent is the _variable_

KEY WORDS
Percent rate
Multiplied
Growth/decay
Exponential
$y = ab^x$
a
b
variable
growing
decaying

Growth/Decay Factors

In an exponential equation, $y = ab^x$, the growth/decay factor is given by the value of ***b***

- If ***b*** > 1, the relation is _growing_
- $y = a(1 + r)^x$
- Growth rate = growth factor – 1

- If 0 < ***b*** < 1, the relation is _decaying_
- $y = a(1 - r)^x$
- Decay rate = 1 – decay factor

EXAMPLE 1 Determine the growth/decay factor or growth/decay rate in each of the following:

a) $A = 500(1.071)^n$	b) $P = 500(0.92)^t$ → 92%	c) $A = 2000(1.045)^n$
Growth factor (b) = 1.071 **Growth rate** = 1.071 – 1 = 0.071 * convert to percent = 0.071 x 100 = 7.1%	b = 0.92 r = 1 – 0.92 = 0.08 8% 8%	b = 1.045 OR 104.5% r = 1.045 – 1 $\boxed{4.5\%}$ = 0.045 = 4.5%

EXAMPLE 2 Which models represent exponential relations? [*Hint: Calculate growth factor by dividing latter y value by the former one*]

- ***If growth /decay factor is constant, then it is exponential***

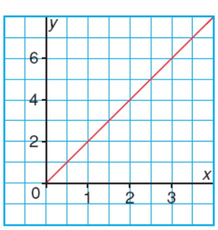
a)

t	*A*	
0	35	
		$\frac{25}{35} = 0.71$
1	25	
		$\frac{15}{25} =$
2	15	
		$\frac{15}{25} =$
3	5	

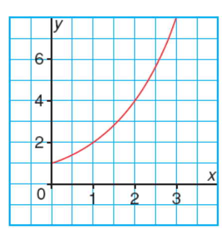
b)

d	*P*	
0	51.2	
1	64	
2	80	
3	100	

c)



d)



e) $y = 10(2)^x$

f) $y = 10x^2$

Practice: Page 315 #2 – 7