GEOMETRIC SERIES

A **geometric series** is the sum of the terms in a geometric sequence.
For example, for the geometric sequence 7, 14, 28, 56 , … , the **geometric series** is 7 + 14 + 28 + 56 + … .
Where t_n represents the value of the n^{th} term, S_n represents the sum of the first n terms.

A series can be calculated in two ways:

❶ $S_n = \frac{a(r^n-1)}{r-1}$

❷ $S_n = \frac{t_{n+1}-t_1}{r-1}$ where $r \neq 1$.

Ex1. For the given geometric series, calculate t_7 and S_7.

5 - 10 + 20 - … $a=5$ $r=\frac{-10}{5}=-2$

$t_n = a(r)^{n-1}$

$t_n = 5(-2)^{n-1}$

$t_7 = 5(-2)^{7-1}$

$= 5(-2)^6$

$= 5(64)$

$\boxed{t_7 = 320}$

$S_7 = \frac{5[(-2)^7-1]}{-2-1} = \frac{5(-129)}{-3}$

$S_7 = 5(43)$

$\boxed{S_7 = 215}$

Ex2. Find the sum of the first 9 terms of the geometric series where the first term is -128 and the sixth is -4.

$S_9 = ?$ $t_1 = -128$ $t_6 = -4$

$t_n = a(r)^{n-1}$

$-4 \rightarrow t_6 = -128(r)^{6-1}$

$-4 = -128(r)^5$

$\frac{1}{32} = r^5$

$\sqrt[5]{2^{-5}} = \sqrt[5]{r^5}$

$r = 2^{-1}$

$\boxed{r = 0.5}$

$S_n = \frac{a(r^n-1)}{r-1}$

$S_9 = \frac{-128[(0.5)^9-1]}{0.5-1}$

$\boxed{S_9 = -255.5}$ ✓

Ex3. Find the sum of the first 10 terms of the geometric series where the 11^{th} term is 3 and the terms decrease by a factor of 1/3.

$S_{10} = ?$ $t_{11} = 3$ $r = 1/3$

$t_n = a(r)^{n-1}$

$3 = a(\frac{1}{3})^{11-1}$

$3 = a(\frac{1}{3})^{10}$

$\frac{3}{3^{-10}} = \frac{a\cancel{3^{-10}}}{\cancel{3^{-10}}}$

$\boxed{a = 3^{11}}$

$S_n = \frac{a(r^n-1)}{r-1}$

$S_{10} = \frac{3^{11}[(3^{-1})^{10}-1]}{\frac{1}{3}-1} = \frac{3^{11}(3^{-10}-1)}{\frac{-2}{3}}$

$= 265716$

Ex4. Calculate the sum of the geometric sequence.

5 - 15 + 45 - … + 3645 $a=5$ $r=-3$

1

$t_n = a(r)^{n-1}$

$3645 = 5(-3)^{n-1}$

$729 = (-3)^{n-1}$

$(-3)^6 = (-3)^{n-1}$

729 | 3
243 | 3
81 | 3
9 | 3
3 | 3
3 | 3
1

$n-1=6$

$\boxed{n=7}$

2

$S_n = \dfrac{a(r^n-1)}{r-1}$

$S_7 = \dfrac{5[(-3)^7-1]}{-3-1}$

$S_7 = 2735$

Ex5. A catering company has 11 customer orders during its first month. For each month afterward, the company has doubled the number of orders than the previous month. How many orders in total did the company fill at the end of its first year?

$t_1 = 11$ $r=2$ $S_{12} = ?$

$S_{12} = \dfrac{a(r^n-1)}{r-1}$

$= \dfrac{11(2^{12}-1)}{2-1}$

$= 45045$

∴ They had 45045 within the first year.

Ex6. At a fish hatchery, fish hatch at different times even though the eggs were all fertilized at the same time. The number of fish that hatched were 2, 10, 50, 250, … , 3906250. if the pattern continues, calculate the total number of fish hatched.

2, 10, 50, 25, …, 3906250 → t_n

$a=2$ $r=5$ $n=10$

$t_n = a(r)^{n-1}$

$3906250 = 2(5)^{n-1}$

$1953125 = 5^{n-1}$

1953125 | 25
78125 | 25
3125 | 25
125 | 25
5 | 5
1

$5^9 = 5^{n-1}$

$n-1=9$

$\boxed{n=10}$

$S_{10} = \dfrac{a(r^n-1)}{r-1} = \dfrac{2(5^{10}-1)}{5-1}$

$S_{10} = 4{,}882{,}812$

∴ 4,882,812 fish hatched.