MAP4C **Date: ____________**

Day 5: Rational Exponents **Unit 6: Algebraic Models**

RECALL:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

CONDITIONS

- m is an integer
- n is a natural number (integer greater than 0)
- a is greater than or equal to 0 if m is even

- ***Radical*** means there is a root $\sqrt{}$
- ***Rational*** means there is an exponent in fraction form

EVALUATING POWERS WITH RATIONAL EXPONENTS

EXAMPLE 1 Rewrite each expression in radical form and then evaluate without a calculator.

a) $81^{\frac{1}{2}}$ $=\sqrt[2]{81}$ $=9$ OR $=(9^2)^{1/2}$ $=9^{2\cdot 1/2}$ $=9$	**b)** $(-64)^{\frac{1}{3}}$ $=\sqrt[3]{-64}$ $=-4$ OR $=(-4^3)^{1/3}$ $=-4^{3\cdot 1/3}$ $=-4$	**c)** $32^{\frac{4}{5}}$ $=\left(\sqrt[5]{32}\right)^4$ $=(2)^4$ $=16$ OR $=(2^5)^{4/5}$ $=2^{5\cdot 4/5}=2^4=16$

SOLVING FOR THE BASE IN A POWER

Rational exponents are useful for solving equations involving powers. For example, take both sides of the equation $x^3 = 8$ to the exponent 1/3 to find the solution x =2.

$(x^3)^{1/3} = (8)^{1/3}$
$x = \sqrt[3]{8}$
$x = 2$

EXAMPLE 2: Solve for the unknown variable, *x*.

a) $x^4 = 16$ $(x^4)^{1/4} = (16)^{1/4}$ $x^{4\cdot 1/4} = \sqrt[4]{16}$ $\boxed{x=2}$	**b)** $x^{\frac{3}{2}} = 27$ $\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = (27)^{2/3}$ $x^{\frac{3}{2}\cdot\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $x = (3)^2$ $x = 9$	**c)** $x^{\frac{2}{3}} = 64$ $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = (64)^{\frac{3}{2}}$ $x = \left(\sqrt[2]{64}\right)^3$ $x = 8^3$ $\underline{x = 512}$

SOLVING A FINANCIAL PROBLEM

EXAMPLE 3: Under annual compounding, an initial investment of \$700 grows to \$900 in 5 years. Determine the annual interest rate, *i*, using the compound interest formula $A = P(1 + i)^n$.

$A = P(1+i)^n$

$900 = 700(1+i)^5$
$\div 700 \quad \div 700$

$(1.2857)^{\frac{1}{5}} = \left[(1+i)^5\right]^{1/5}$

$1.0515 = 1+i$

$i \xrightarrow{+1} \xrightarrow{\wedge 5} \xrightarrow{\times 700} 900$

③ -1 ② $\wedge\frac{1}{5}$ ① $\div 700$

$1.0515 = 1 + i$
$-1 \quad -1$
$0.0515 = i$

$\boxed{i = 5.15\%}$

COMPLETE: Page 376 #3ad, 5cd, 6acf, 9, 10ade, 12b, 13, 14ac