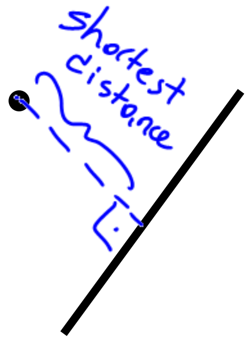
Shortest Distance from a Point to a Line



- Given the point we can draw infinite different lines to the line but...
- The shortest distance is the line that hits it at a 90°
- The shortest distance from a point to a line is the perpendicular distance from the point to your line.

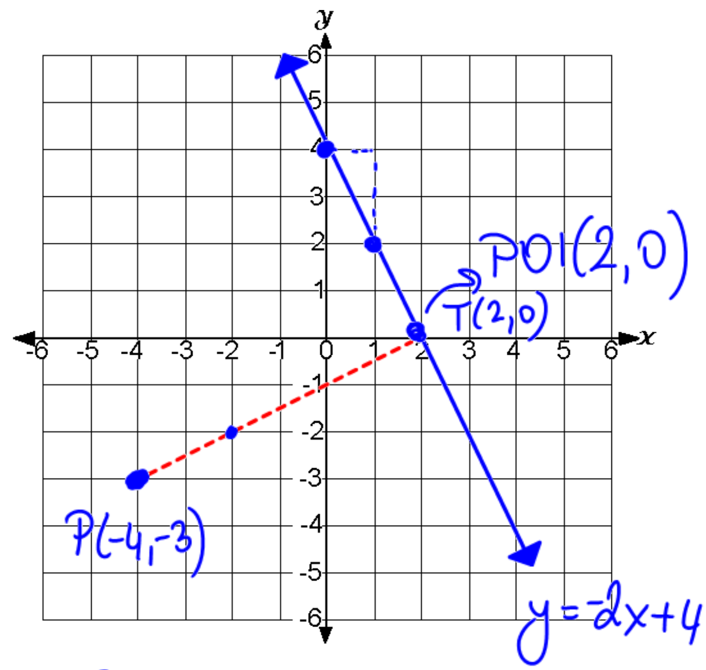
METHOD 1: Finding the Shortest Distance Graphically

Ex1. Find the shortest distance **graphically** between the point P $(-4,-3)$ and the equation $y = -2x + 4$.

Step 1: Graph $y = -2x + 4$

$m = \frac{-2}{1}$ y-int

$\boxed{m_{\text{shortest distance}} = \frac{1}{2}}$



reads line segment PT

T(2,0)

P(-4,-3)

$\overline{PT} = \sqrt{(-4-2)^2 + (-3-0)^2}$

$= \sqrt{(-6)^2 + (-3)^2}$

$(-6)^2 = (-6)(-6) = 36$

$(-3)^2 = (-3)(-3) = 9$

$= \sqrt{36 + 9}$

$= \sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

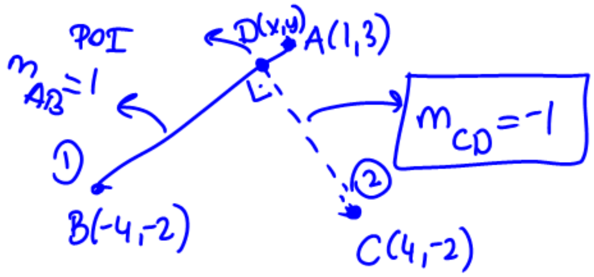
perfect square

exact form

METHOD 2: Finding the Shortest Distance Algebraically

Ex2. Find the shortest distance from the point $C(4,-2)$ to the line passing through the points $A(1,3)$ and $B(-4,-2)$.

Step 1 Find equation of the line AB.
Step 2 Draw a line perpendicular to AB that goes through C. Let the point on AB be called D. Find the equation of the line CD.
Step 3 Find D, the POI of AB and CD (substitution or elimination).
Step 4 Find the length of CD.



shortest distance

Step 1: Equation $\overline{AB}$

$A(1,3)$ $B(-4,-2)$

$m_{AB} = \frac{3-(-2)}{1-(-4)} = \frac{3+2}{1+4} = \frac{5}{5}$

$\boxed{m_{AB}=1}$ $A(1,3)$

$y = m(x-p)+q$

$y = 1(x-1)+3$

$y = x-1+3$

① $\boxed{y = x+2}$

Step 2: Equation of $\overline{CD}$

$m_{CD}=-1$ $C(4,-2)$

$y = m(x-p)+q$

$y = -1(x-4)+(-2)$

$y = -x+4-2$

② $\boxed{y=-x+2}$

Step 3: Find POI D(x,y)

① $y = x+2$

② $y = -x+2$

Sub ① → ②

$x+2 = -x+2$

$x+x = 2-2$

$\frac{2x}{2} = \frac{0}{2}$

$\boxed{x=0}$

$y = x+2$

$y = 0+2$

$\boxed{y=2}$

POI ⇒ D(0,2)

Step 4: Shortest distance

$C(4,-2)$ $D(0,2)$

$\overline{CD} = \sqrt{(4-0)^2+(-2-2)^2}$

$= \sqrt{(4)^2+(-4)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$= \sqrt{16 \cdot 2}$

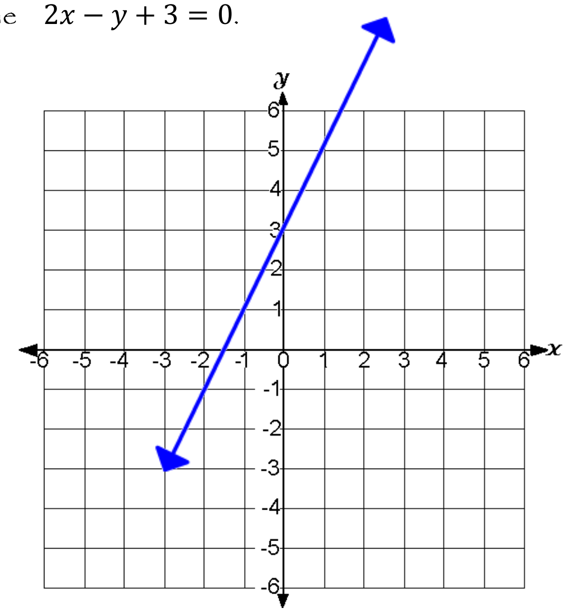
$= \sqrt{16} \cdot \sqrt{2}$

$= 4\sqrt{2}$

∴ The shortest distance from C to $\overline{AB}$ is $\sqrt{32}$ or $4\sqrt{2}$

Practice

Ex3. Determine the shortest distance **graphically** from $A(3,-1)$ to the line $2x - y + 3 = 0$.



Ex4: **Algebraically** determine the shortest distance from the point $P(5,2)$ to the line passing through the points $R(-6,4)$ and $S(-2,-4)$.

Ex 5. Triangle ABC has vertices $A(3,4)$, $B(-5,2)$, and $C(1,-4)$. Determine an equation for AE, the **altitude** from A to BC. What is the area of triangle ABC?

An altitude of a triangle is ____.