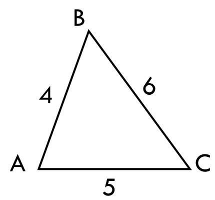
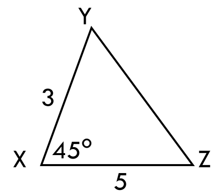
THE COSINE LAW

We can use the cosine law when we know:

SIDE - SIDE - SIDE (SSS)



SIDE - ANGLE - SIDE (SAS)



KEY WORDS
Length
Two sides
Opposite angle

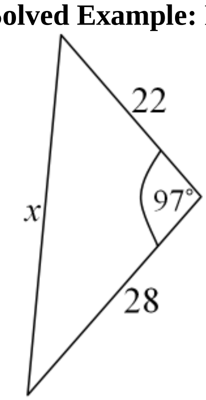
THE COSINE LAW: SOLVING FOR SIDES

If you need to find the length of a side, you need to know the other two sides and the opposite angles
You need to use the version of the Cosine Rule where a^2 is the subject of the formula:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

Side a is the one you are trying to find. Sides b and c are the other sides, and angle A is the angle opposite to side a.

Solved Example: Determine the length of x.



Step 1 Start by writing out the Cosine Rule formula for finding sides:

$a^2 = b^2 + c^2 - 2bc\cos(A)$

Step 2 Fill in the values you know, and the unknown length:

$x^2 = 22^2 + 28^2 - 2\times22\times28\times\cos(97°)$

It doesn't matter which way around you put sides *b* and *c* – it will work both ways.

Step 3 Evaluate the right-hand-side and then square-root to find the length:

$x^2 = 22^2 + 28^2 - 2\times22\times28\times\cos(97°)$ *(evaluate the right hand side)*

$x^2 = 1418.143.....$ *(square-root both sides)*

$x = 37.7$ (accurate to 3 significant figures)

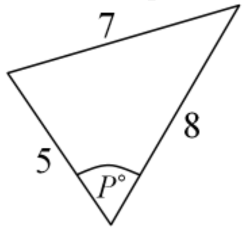
THE COSINE LAW: SOLVING FOR AN ANGLE

If you need to find the size of an angle, you need to use the version of the Cosine Rule where the cos(A) is on the left:

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

It is very important to get the terms on the top in the correct order; b and c are either side of angle A which you are trying to find and these can be either way around, but side a must be the side opposite angle A.

Solved example: Determine the angle of P to the nearest degree.



Step 1 Start by writing out the Cosine Rule formula for finding angles:

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

Step 2 Fill in the values you know, and the unknown length:

$$\cos(P°) = \frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8}$$

Remember to make sure that the terms on top of the fraction are in the correct order.

Step 3 Evaluate the right-hand-side and then use inverse-cosine ($\cos^{-1}$) to find the angle:

$$\cos(P°) = \frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8}$$ *(evaluate the right-hand side)*

$$\cos(P°) = 0.5$$ *(do the inverse-cosine of both sides)*

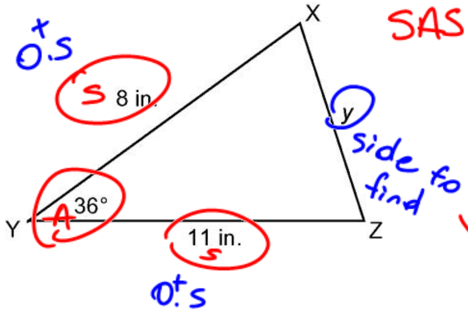
$$P° = \cos^{-1}(0.5) = 60°$$

The **COSINE LAW** can be used to solve triangles when given information includes:

- SIDE – SIDE – SIDE
- TWO SIDE MEASURES AND AN ANGLE IN BETWEEN THOSE SIDES.

EXAMPLES:

1. Find the measure of the unknown side to the nearest tenth



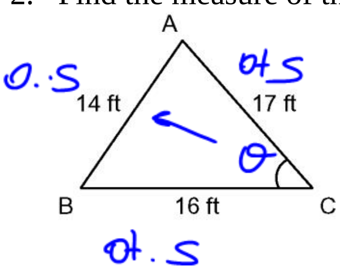
$y^2 = 8^2 + 11^2 - 2 \cdot 8 \cdot 11 \cdot \cos 36°$

$\sqrt{y^2} = \sqrt{42.6130}$ → keep 4 dec. places

$y \doteq 6.5$ in.

∴ Side y is 6.5 in.

2. Find the measure of the marked angle to the nearest degree



$\cos\theta = \frac{16^2 + 17^2 - 14^2}{2 \cdot 16 \cdot 17}$

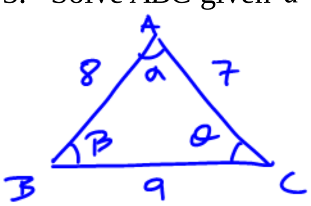
$\cos\theta = \frac{349}{544}$

$\cos^{-1}\left(\frac{349}{544}\right) = \theta$

$\theta \doteq 50°$

∴ ∠C is 50°

3. Solve *ABC* given $a = 9$ cm, $b = 7$ cm, and $c = 8$ cm



Step 1: solve α using cosine law

$\cos\alpha = \frac{8^2 + 7^2 - 9^2}{2 \cdot 8 \cdot 7}$

$\cos\alpha = \frac{32}{112}$

$\cos^{-1}\left(\frac{32}{112}\right) = \alpha$

$\alpha \doteq 73.4°$

Step 2: solve for θ using sine law

$\frac{\sin\theta}{8} = \frac{\sin 73.4}{9}$

$\sin\theta = \frac{8 \times \sin 73.4}{9}$

$\sin^{-1}\left(\frac{8 \times \sin 73.4}{9}\right) = \theta$

$\theta \doteq 58.4°$

Step 3: Solve for β

$\alpha + \theta + \beta = 180$

$73.4 + 58.4 + \beta = 180$

$\beta = 180 - 73.4 - 58.4$

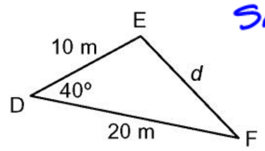
$\beta = 48.2°$

∴ ∠A is 73.4°
∠B is 48.2°
∠C is 58.4°

Cosine Law Practice

1. Find the measure of the unknown side to the nearest tenth.

a) SAS



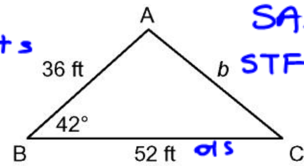
$d^2 = 10^2 + 20^2 - 2 \cdot 10 \cdot 20 \cdot \cos 40^\circ$

$d^2 = 193.5822$

$d = 13.9\text{ m}$

∴ Side d is 13.9 m.

b) SAS



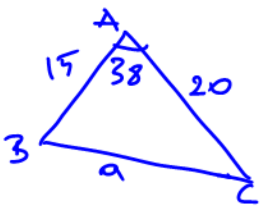
$b^2 = 36^2 + 52^2 - 2 \cdot 36 \cdot 52 \cdot \cos 42$

$b^2 = 1217.6658$

$b = 34.9\text{ ft}$

∴ side b is 34.9 ft

c) In ABC given that $\angle A = 38^\circ$, $b = 20$ m, and $c = 15$ m, find a



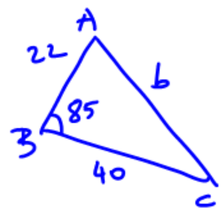
$a^2 = 15^2 + 20^2 - 2 \cdot 15 \cdot 20 \cdot \cos 38$

$a^2 = 152.1935$

$a \doteq 12.3$

∴ side a is 12.3 m

d) In ABC given that $\angle B = 85^\circ$, $a = 40$ mm, and $c = 22$ mm, find b



$b^2 = 22^2 + 40^2 - 2 \cdot 22 \cdot 40 \cdot \cos 85$

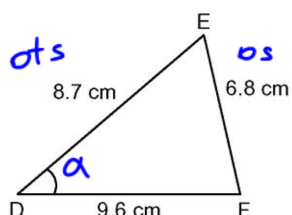
$b^2 = 1930.6059$

$b \doteq 43.9\text{ m}$

∴ Side b is 43.9 mm.

2. Find the measure of the marked angle to the nearest degree.

a)



$\cos\alpha = \dfrac{(8.7)^2 + (9.6)^2 - (6.8)^2}{2 \cdot (8.7) \cdot (9.6)}$

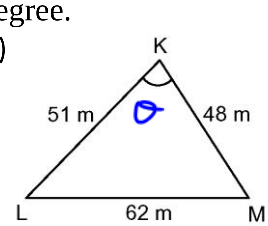
$\cos\alpha = \dfrac{121.61}{167.04}$

$\cos^{-1}\left(\dfrac{121.61}{167.04}\right) = \alpha$

$\alpha = 43^\circ$

∴ ∠D is 43°

b)



$\cos\theta = \dfrac{(51)^2 + (48)^2 - (62)^2}{2 \cdot 51 \cdot 48}$

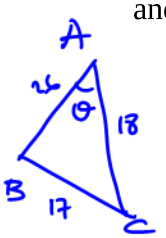
$\cos\theta = \dfrac{1061}{4896}$

$\cos^{-1}\left(\dfrac{1061}{4896}\right) = \theta$

$\theta \doteq 77^\circ$

∴ ∠K is 77°

c) In ABC given that $a = 17$ cm, $b = 18$ cm, and $c = 26$ cm, find $\angle A$



$\cos\theta = \dfrac{26^2 + 18^2 - 17^2}{2 \cdot 26 \cdot 18}$

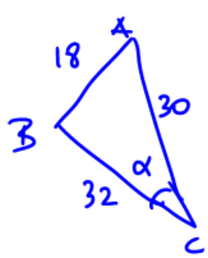
$\cos\theta = \dfrac{711}{936}$

$\cos^{-1}\left(\dfrac{711}{936}\right) = \theta$

$\theta \doteq 40.6^\circ$

∴ ∠A is 40.6°

d) In ABC given that $a = 32$ ft, $b = 30$ ft, and $c = 18$ ft, find $\angle C$



$\cos\alpha = \dfrac{32^2 + 30^2 - 18^2}{2 \cdot 32 \cdot 30}$

$\cos\alpha = \dfrac{1600}{1920}$

$\cos^{-1}\left(\dfrac{1600}{1920}\right) = \alpha$

$\alpha \doteq 33.6$

∴ ∠C is 33.6°

3. <u>Solve</u> *ABC* given $\angle A = 52^\circ$, $b = 26$ cm, $c = 18$ cm

4. <u>Solve</u> *ABC* given $a = 16$ cm, $b = 19$ cm, $c = 11$ cm

5. Mark says he can solve the following triangle using the sine law. Cassandra says she can solve the triangle using the cosine law. Who is correct? Explain.