THE COSINE LAW

We can use the cosine law when we know:

SIDE - SIDE - SIDE (SSS)





SIDE - ANGLE - SIDE (SAS)

KEY WORDS Length Two sides Opposite angle

THE COSINE LAW: SOLVING FOR SIDES

Solved Example: Determine the length of x.

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If you need to find the <u>length</u> of a side, you need to know the other <u>two</u> sides and the opposite angles You need to use the version of the Cosine Rule where a^2 is the subject of the formula:

$a^2 = b^2 + c^2 - 2bc \cos(A)$

Side a is the one you are trying to find. Sides b and c are the other sides, and angle A is the angle opposite to side a.

Step 1 Start by writing out the Cosine Rule formula for finding sides:

 $a^2 = b^2 + c^2 - 2bc \cos(A)$

 $x^2 = 1418.143....$

Step 2 Fill in the values you know, and the unknown length:

 $x^2 = 22^2 + 28^2 - 2 \times 22 \times 28 \times \cos(97^\circ)$

It doesn't matter which way around you put sides b and c – it will work both ways.

Step 3 Evaluate the right-hand-side and then square-root to find the length:

 $x^2 = 22^2 + 28^2 - 2 \times 22 \times 28 \times \cos(97^\circ)$ (evaluate the right hand side)

(square-root both sides)

x = 37.7 (accurate to 3 significant figures)

THE COSINE LAW: SOLVING FOR AN ANGLE

If you need to find the size of an angle, you need to use the version of the Cosine Rule where the cos(A) is on the left:

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

It is very important to get the terms on the top in the correct order; b and c are either side of angle A which you are trying to find and these can be either way around, but side a must be the side opposite angle A.

Solved example: Determine the angle of P to the nearest degree.

Step 1 Start by writing out the Cosine Rule formula for finding angles:

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

Step 2 Fill in the values you know, and the unknown length:

$$\cos(P^{\circ}) = \frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8}$$

Remember to make sure that the terms on top of the fraction are in the correct order.

Step 3 Evaluate the right-hand-side and then use inverse-cosine (\cos^{-1}) to find the angle:

 $\cos(P^{\circ}) = \frac{5^{2} + 8^{2} - 7^{2}}{2 \times 5 \times 8}$ (evaluate the right-hand side) $\cos(P^{\circ}) = 0.5$ (do the inverse-cosine of both sides) $P^{\circ} = \cos^{-1}(0.5) = 60^{\circ}$

The **COSINE LAW** can be used to solve triangles when given information includes:

- SIDE SIDE SIDE
- TWO SIDE MEASURES AND AN ANGLE IN BETWEEN THOSE SIDES.

EXAMPLES:

1. Find the measure of the unknown side to the nearest tenth



2. Find the measure of the marked angle to the nearest degree



3. Solve *ABC* given a = 9 cm, b = 7 cm, and c = 8 cm

$$\frac{8}{3} \stackrel{\circ}{\Theta} \stackrel{\tau}{T} \xrightarrow{\text{Shepl}: \text{ solve } d \text{ using } \text{ using sine } law}{\text{cosine } law} \qquad \left| \begin{array}{c} \underline{\text{Shepl}: \text{ solve } \text{ for } \Theta} \\ using \text{ sine } law \\ \underline{\text{using sine } law} \\ \frac{3}{9} \stackrel{\circ}{\Theta} \stackrel{\tau}{C} \xrightarrow{\text{Cos} \alpha} = \frac{8^2 + 7^2 - 9^2}{2 \cdot 8 \cdot 7} \\ \frac{3}{2 \cdot 7} \\ \frac{3}{2 \cdot 7} \\ \frac{3}{2 \cdot 7$$

Cosine Law Practice

1. Find the measure of the unknown side to the nearest tenth.

a)
$$\int_{0}^{\infty} \int_{0}^{\infty} \int$$

3. Solve ABC given $\angle A = 52^{\circ}$, b = 26 cm, c = 18 cm

4. Solve ABC given
$$a = 16$$
 cm, $b = 19$ cm, $c = 11$ cm
 $19 + 16^{2} + 1$

5. Mark says he can solve the following triangle using the sine law. Cassandra says she can solve the triangle using the cosine law. Who is correct? Explain.



Cassy is correct because there is not enough information for the sinc low. The condition (553) matches the casine low formule.