$\qquad$

## THE COSINE LAW

We can use the cosine law when we know:

SIDE - SIDE - SIDE (SSS)


SIDE - ANGLE - SIDE (SAS)


KEY WORDS
Length
Two sides
Opposite angle

## THE COSINE LAW: SOLVING FOR SIDES

If you need to find the length of a side, you need to know the other two sides and the opposite ongle. You need to use the version of the Cosine Rule where $\mathrm{a}^{2}$ is the subject of the formula:

$$
\text { side to find } a^{a^{2}=b^{2}+c^{2}-2 b c} \cos (\underbrace{A)}_{\text {other sides }} \text { opposite angle to side to find }
$$

Side a is the one you are trying to find. Sides b and c are the other sides, and angle $A$ is the angle opposite to side a.

$\qquad$
Day 5: The Cosine Law

## THE COSINE LAW: SOLVING FOR AN ANGLE

If you need to find the size of an angle, you need to use the version of the Cosine Rule where the $\cos (\mathrm{A})$ is on the left:

$$
\cos (A)=\frac{b^{2}+c^{2}-a^{2}}{2 b c}
$$

It is very important to get the terms on the top in the correct order; b and c are either side of angle A which you are trying to find and these can be either way around, but side a must be the side opposite angle A.

Solved example: Determine the angle of P to the nearest degree.


The COSINE LAW can be used to solve triangles when given information includes:

- SIDE - SIDE - SIDE
- TWO SIDE MEASURES AND AN ANGLE IN BETWEEN THOSE SIDES.

EXAMPLES:

1. Find the measure of the unknown side to the nearest tenth

2. Find the measure of the marked angle to the nearest degree


$$
\begin{aligned}
& \cos A=\frac{b^{2}+c^{2}-a^{2}}{2 \cdot b \cdot c} \text { eras the labels from the } A \text {, because } \\
& \cos x=\frac{17^{2}+16^{2}-14^{2}}{2 \cdot 17 \cdot 16} \\
& \cos x=\frac{349}{544} \\
& \cos x=0.6415 \\
& \cos ^{-1}(0.6415)=x \\
& x=50
\end{aligned}
$$

3. Solve $A B C$ given $a=9 \mathrm{~cm}, b=7 \mathrm{~cm}$, and $c=8 \mathrm{~cm}$


$$
\begin{array}{cr}
\cos A=\frac{8^{-2}+7^{2}-9^{2}}{2 \cdot 8.7} & \frac{\sin B}{7}=\frac{\sin 73}{9} \\
\therefore \cos A=\frac{32}{112} & \sin B=\frac{\sin 73}{9} .7 \\
\cos A=0.2857 & \sin B=0.7438 \\
\cos ^{-1}(0.2857)=A & \sin ^{-1}(0.7438)=B \\
A=73^{\circ} & B=48
\end{array}
$$

$$
\left.\begin{array}{cc}
\frac{\sin B}{7}=\frac{\sin 73}{9} & \angle C=180-48-73 \\
\sin B=\frac{\sin 73.7}{9} & \angle C=59 \\
\sin B=0.7438 & \therefore \angle A=73
\end{array}\right) a=9 .
$$

B

Cosine Law Practice

1. Find the measure of the unknown side to the nearest tenth.
a)


$$
\begin{aligned}
d^{2} & =10^{2}+20^{2}-2 \cdot 10 \cdot 20 \cdot \cos 40 \\
d^{2} & =193.5822 \quad \sqrt{\text { both sides }} \\
\sqrt{d^{2}} & =\sqrt{193.5822} \\
d & =13.9 \mathrm{~m}
\end{aligned}
$$

c) In $A B C$ given that $\angle A=38^{\circ}, b=20 \mathrm{~m}$, and $c=15 \mathrm{~m}$, find $a$

$$
\begin{aligned}
& a^{2}=20^{2}+15^{2}-2 \cdot 20 \cdot 15 \cdot \cos 38 \\
& a^{2}=152.1935 \\
& \sqrt{a^{2}}=\sqrt{152.1935} \\
& a=12
\end{aligned}
$$

b)


$$
\sqrt{6^{2}}=\sqrt{1217.6658}
$$

$$
b \doteq 34.9
$$

d) In $A B C$ given that $\angle B=85^{\circ}$, $a=40 \mathrm{~mm}$, and $c=22 \mathrm{~mm}$, find $b$

$$
\begin{aligned}
b^{2} & =40^{2}+22^{2}-2 \cdot 40 \cdot 22 \cdot \cos 85 \\
b^{2} & =1930 \cdot 6059 \\
\sqrt{b^{2}} & =1930.6059 \\
b & =44
\end{aligned}
$$

2. Find the measure of the marked angle to the nearest degree.
a)


$$
\begin{aligned}
& \cos A=\frac{8.7^{2}+9.6^{2}-6.8^{2}}{2 \cdot 8.7 \cdot 9.6} \\
& \cos A=\frac{121.61}{167.04} \\
& \cos A=0.7280 \\
& \cos ^{-1}(0.7280)=A \\
& A=43^{\circ}
\end{aligned}
$$

c) In $A B C$ given that $a=17 \mathrm{~cm}, b=18 \mathrm{~cm}$, and $c=26 \mathrm{~cm}$, find $\angle A$

$$
\begin{gathered}
\cos A=\frac{18^{2}+26^{2}-17^{2}}{2 \cdot 18.26} \\
\cos A=\frac{71}{936} \\
\cos ^{-1}(71 / 936)=A \\
A=86^{\circ}
\end{gathered}
$$



$$
\begin{gathered}
\cos A=\frac{51^{2}+48^{2}-62^{2}}{2 \cdot 51 \cdot 48} \\
\cos A=\frac{1061}{4896} \\
\cos ^{-1}(1061 / 4896)=A \\
A=77
\end{gathered}
$$

d) In $A B C$ given that $a=32 \mathrm{ft}, b=30 \mathrm{ft}$, and $c=18 \mathrm{ft}$, find $\angle C$

$$
\begin{gathered}
\cos C=\frac{32^{2}+30^{2}-18^{2}}{2 \cdot 32 \cdot 30} \\
\cos C=\frac{1600}{1920} \\
\cos ^{-1}\left(\frac{1600}{1920}\right)=C \\
C \doteq 34^{\circ}
\end{gathered}
$$

$\qquad$
3. Solve $A B C$ given $\angle A=52^{\circ}, b=26 \mathrm{~cm}, c=18 \mathrm{~cm}$


$$
a^{2}=18^{2}+26^{2}-2 \cdot 18 \cdot 26 \cdot \cos 52
$$

$$
\lambda^{2}=423.7409
$$

$$
\sqrt{a_{1}^{2}}=\sqrt{423.7409}
$$

$$
a \doteq 21
$$



$$
\begin{aligned}
& \frac{\sin B}{26}=\frac{\sin 52}{21} \\
& \sin B=\frac{\sin 52}{21} \cdot 26 \\
& \sin B=0.9756 \\
& \sin ^{-1}(0.9756)=B \\
& \quad B \pm 77^{\circ}
\end{aligned}
$$

Step 3 $\angle C=180-52-77$

$$
\angle C=51
$$

4. Solve $A B C$ given $a=16 \mathrm{~cm}, b=19 \mathrm{~cm}, c=11 \mathrm{~cm}$


$$
\begin{gathered}
\cos A=\frac{19^{2}+11^{2}-16^{2}}{2 \cdot 19 \cdot 11} \\
\cos A=\frac{226}{418} \\
\cos ^{-1}(226 / 418)=A \\
A=57^{\circ}
\end{gathered}
$$



$$
\begin{aligned}
& \quad \frac{\sin B}{19}=\frac{\sin 57}{16} \\
& \sin B=\frac{\sin 57}{16} \cdot 19 \\
& \sin B=0.9959 \quad \therefore \angle A=57 a=16 \\
& \sin ^{-1}\left(0.995^{\circ}\right)=B \\
& \left(B=85^{\circ}\right.
\end{aligned} \quad \angle C=38 \quad b=19.11
$$

Step 3:

$$
\begin{aligned}
& \angle C=180-57-85 \\
& \angle C=38^{\circ}
\end{aligned}
$$

5. Mark says he can solve the following triangle using the sine law. Cassandra says she can solve the triangle using the cosine law. Who is correct? Explain.


To be able to use the sine $l 0, w^{\text {or }}$, we need tho meabwenent of a side.
In this triangle, we do not hove any side leapt. Thus, we cannot solve this triopl.

$$
\begin{aligned}
& \therefore \angle A=52^{\circ} \quad a=21 \mathrm{~cm} \\
& \angle B=77^{\circ} \\
& b=26 \mathrm{~cm} \\
& \angle C=51^{\circ} \quad C=18 \mathrm{~cm}
\end{aligned}
$$

