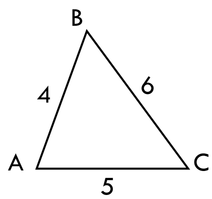
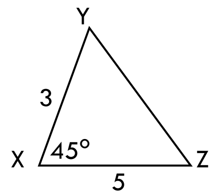
THE COSINE LAW

We can use the cosine law when we know:

SIDE - SIDE - SIDE (SSS)	**SIDE - ANGLE - SIDE (SAS)**
	

KEY WORDS
Length
Two sides
Opposite angle

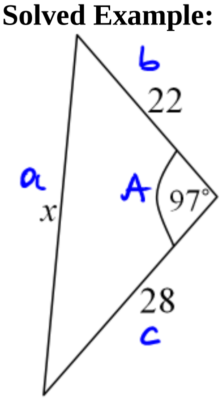
THE COSINE LAW: SOLVING FOR SIDES

If you need to find the length of a side, you need to know the other two sides and the opposite angle. You need to use the version of the Cosine Rule where a^2 is the subject of the formula:

side to find ← $$a^2 = \underbrace{b^2 + c^2 - 2bc}_{\text{other sides}}\cos(A)$$ → opposite angle to side to find

Side a is the one you are trying to find. Sides b and c are the other sides, and angle A is the angle opposite to side a.

Solved Example: Determine the length of x.



Step 1: Write the formula

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Step 2: Label the triangle, set up your equation

$$x^2 = 22^2 + 28^2 - 2 \cdot 22 \cdot 28 \cdot \cos 97$$ (input RS into your calculator in one step)

$$x^2 = 1418.143031$$ square root both sides

$$\sqrt{x^2} = \sqrt{1418.143031}$$

$$x = 37.6582$$

∴ x is approximately 38 units.

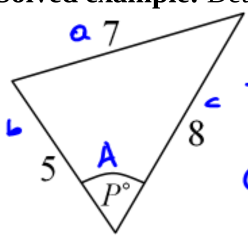
THE COSINE LAW: SOLVING FOR AN ANGLE

If you need to find the size of an angle, you need to use the version of the Cosine Rule where the cos(A) is on the left:

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

It is very important to get the terms on the top in the correct order; b and c are either side of angle A which you are trying to find and these can be either way around, but side a must be the side opposite angle A.

Solved example: Determine the angle of P to the nearest degree.



Step1: Write the formula

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

Step2: Label the triangle based on the formula
Set up the equation

$$\cos P = \frac{5^2+8^2-7^2}{2\cdot 5\cdot 8}$$

simplify numerator and denominator independently.

$$\cos P = \frac{40}{80}$$

$$\cos P = 0.5$$

$$\cos^{-1}(0.5) = P$$

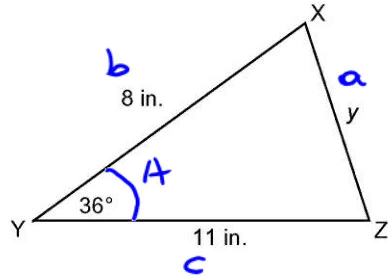
$$\boxed{P = 60^\circ}$$

The **COSINE LAW** can be used to solve triangles when given information includes:

- SIDE – SIDE – SIDE
- TWO SIDE MEASURES AND AN ANGLE IN BETWEEN THOSE SIDES.

EXAMPLES:

1. Find the measure of the unknown side to the nearest tenth



$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos A$

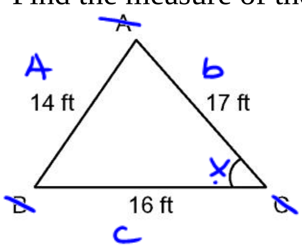
$y^2 = 8^2 + 11^2 - 2 \cdot 8 \cdot 11 \cdot \cos 36$

$y^2 = 42.6130$ $\sqrt{\ }$ both sides

$\sqrt{y^2} = \sqrt{42.613}$

$y \doteq 6.5$

2. Find the measure of the marked angle to the nearest degree



$\cos A = \dfrac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$ erase the labels from the Δ, because we will label it based on the formula

$\cos X = \dfrac{17^2 + 16^2 - 14^2}{2 \cdot 17 \cdot 16}$

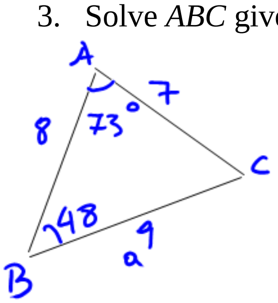
$\cos X = \dfrac{349}{544}$

$\cos x = 0.6415$

$\cos^{-1}(0.6415) = x$

$\boxed{X = 50}$

3. Solve ABC given $a = 9$ cm, $b = 7$ cm, and $c = 8$ cm



$\cos A = \dfrac{8^2 + 7^2 - 9^2}{2 \cdot 8 \cdot 7}$

$\cos A = \dfrac{32}{112}$

$\cos A = 0.2857$

$\cos^{-1}(0.2857) = A$

$\boxed{A = 73^\circ}$

$\dfrac{\sin B}{7} = \dfrac{\sin 73}{9}$

$\sin B = \dfrac{\sin 73}{9} \cdot 7$

$\sin B = 0.7438$

$\sin^{-1}(0.7438) = B$

$\boxed{B = 48}$

$\angle C = 180 - 48 - 73$

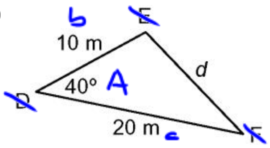
$\angle C = 59$

$\therefore \angle A = 73$ $a = 9$
$\angle B = 48$ $b = 7$
$\angle C = 59$ $c = 8$

Cosine Law Practice

1. Find the measure of the unknown side to the nearest tenth.

a)



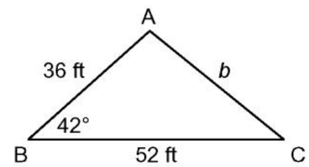
$d^2 = 10^2 + 20^2 - 2 \cdot 10 \cdot 20 \cdot \cos 40$

$d^2 = 193.5822$ $\sqrt{\ }$ both sides

$\sqrt{d^2} = \sqrt{193.5822}$

$\boxed{d = 13.9\text{m}}$

b)



$b^2 = 36^2 + 52^2 - 2 \cdot 36 \cdot 52 \cdot \cos 42$

$b^2 = 1217.6658$

$\sqrt{b^2} = \sqrt{1217.6658}$

$\boxed{b \doteq 34.9}$

c) In *ABC* given that $\angle A = 38°$, $b = 20$ m, and $c = 15$ m, find *a*

$a^2 = 20^2 + 15^2 - 2 \cdot 20 \cdot 15 \cdot \cos 38$

$a^2 = 152.1935$

$\sqrt{a^2} = \sqrt{152.1935}$

$\boxed{a \doteq 12}$

d) In *ABC* given that $\angle B = 85°$, $a = 40$ mm, and $c = 22$ mm, find *b*

$b^2 = 40^2 + 22^2 - 2 \cdot 40 \cdot 22 \cdot \cos 85$

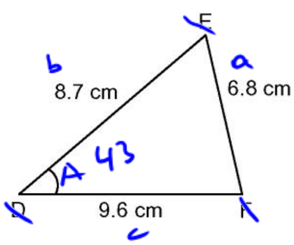
$b^2 = 1930.6059$

$\sqrt{b^2} = \sqrt{1930.6059}$

$\boxed{b \doteq 44}$

2. Find the measure of the marked angle to the nearest degree.

a)



$\cos A = \dfrac{8.7^2 + 9.6^2 - 6.8^2}{2 \cdot 8.7 \cdot 9.6}$

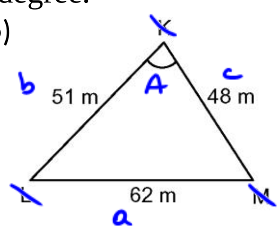
$\cos A = \dfrac{121.61}{167.04}$

$\cos A = 0.7280$

$\cos^{-1}(0.7280) = A$

$\boxed{A = 43°}$

b)



$\cos A = \dfrac{51^2 + 48^2 - 62^2}{2 \cdot 51 \cdot 48}$

$\cos A = \dfrac{1061}{4896}$

$\cos^{-1}(1061/4896) = A$

$\boxed{A \doteq 77°}$

c) In *ABC* given that $a = 17$ cm, $b = 18$ cm, and $c = 26$ cm, find $\angle A$

$\cos A = \dfrac{18^2 + 26^2 - 17^2}{2 \cdot 18 \cdot 26}$

$\cos A = \dfrac{71}{936}$

$\cos^{-1}(71/936) = A$

$\boxed{A \doteq 86°}$

d) In *ABC* given that $a = 32$ ft, $b = 30$ ft, and $c = 18$ ft, find $\angle C$

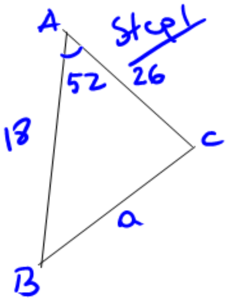
$\cos C = \dfrac{32^2 + 30^2 - 18^2}{2 \cdot 32 \cdot 30}$

$\cos C = \dfrac{1600}{1920}$

$\cos^{-1}(1600/1920) = C$

$\boxed{C \doteq 34°}$

3. Solve ABC given $\angle A = 52°$, $b = 26$ cm, $c = 18$ cm



Step1

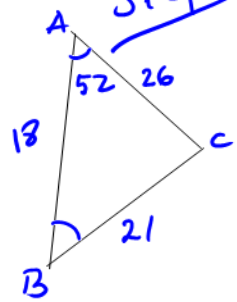
$a^2 = 18^2 + 26^2 - 2 \cdot 18 \cdot 26 \cdot \cos 52$

$a^2 = 423.7409$

$\sqrt{a^2} = \sqrt{423.7409}$

$\boxed{a \doteq 21}$

Step2



$\frac{\sin B}{26} = \frac{\sin 52}{21}$

$\sin B = \frac{\sin 52}{21} \cdot 26$

$\sin B = 0.9756$

$\sin^{-1}(0.9756) = B$

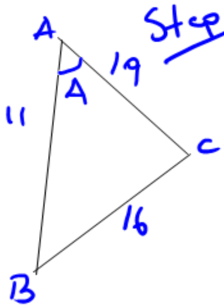
$\boxed{B \doteq 77°}$

Step3 $\angle C = 180 - 52 - 77$

$\angle C = 51$

$\therefore \angle A = 52°$ $a = 21$ cm
$\angle B = 77°$ $b = 26$ cm
$\angle C = 51°$ $c = 18$ cm

4. Solve ABC given $a = 16$ cm, $b = 19$ cm, $c = 11$ cm



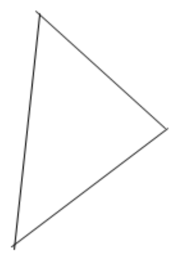
Step1

$\cos A = \frac{19^2 + 11^2 - 16^2}{2 \cdot 19 \cdot 11}$

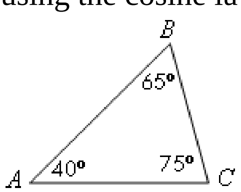
$\cos A = \frac{226}{418}$

$\cos^{-1}(226/418) = A$

$\boxed{A = 57°}$



5. Mark says he can solve the following triangle using the sine law. Cassandra says she can solve the triangle using the cosine law. Who is correct? Explain.



COMPLETE: p. 38 #3b, 4bc, 7c, 8-10, 17