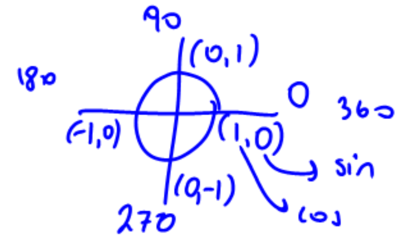
Stretches of Sinusoidal Functions

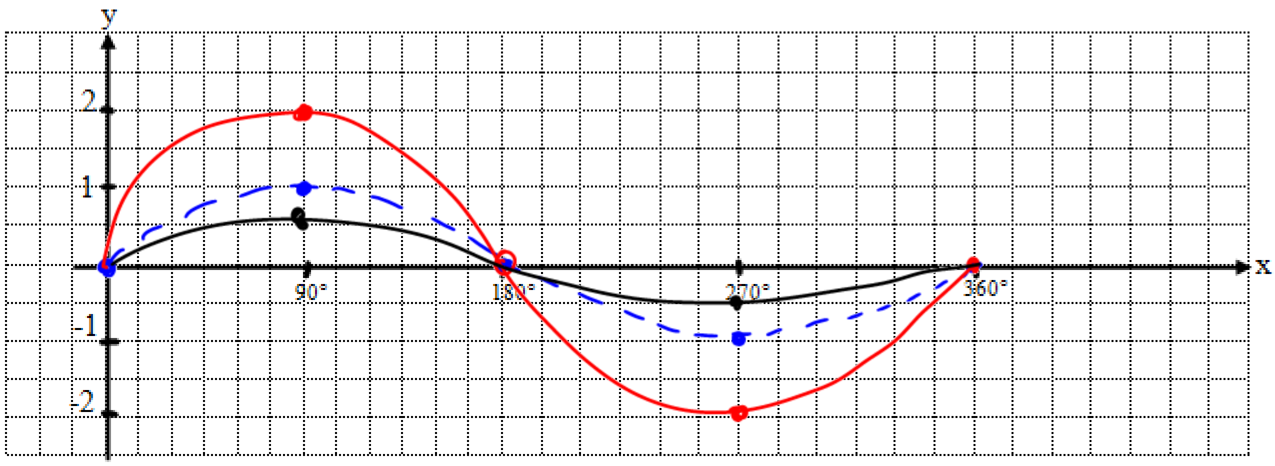
$$f(x) = a\sin[k(x-d)] + c \text{ and } f(x) = a\cos[k(x-d)] + c$$

Vertical Stretches: Investigating for *a*

Recall: $y = af(x)$ is the image of $y = f(x)$ under a transformation which causes a **vertical stretch**.
Example 1: Graph $y = \sin\theta$ and $y = 2\sin\theta$, for $0° \le \theta \le 360°$.

θ	$0°$	$90°$	$180°$	$270°$	$360°$
$y = \sin\theta$	0	1	0	-1	0
$y = 2\sin\theta$	0	2	0	-2	0





For $y = 2\sin\theta$,
1. What coordinate is affected? y coordinate

2. What points are unaffected (invariant)? 0, 180, 360°

3. What is amplitude, a, of the function? 2

4. What is the period? 360

5. What is the equation of the axis of the curve? $\frac{max+min}{2} = \frac{2+(-2)}{2} = 0$ $y=0$

6. State the domain and range. D: $\{x \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid -2 \le y \le 2\}$

Example 2: Graph $y = \frac{1}{2}\sin\theta$, for $0° \le \theta \le 360°$ on the above grid.

θ	$0°$	$90°$	$180°$	$270°$	$360°$
$y = \frac{1}{2}\sin\theta$	0	0.5	0	-0.5	0

SUMMARY,
For $a > 1$, the graph is **stretched** vertically (expanded) by a factor of a.
For $0 < a < 1$, the graph is **compressed** vertically by a factor of a.

a= amplitude

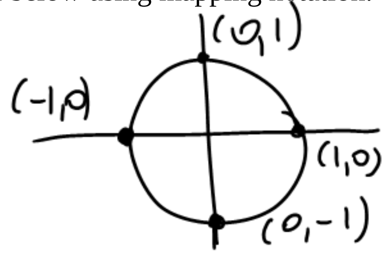
The amplitude of each function $y = a\sin\theta$ and $y = a\cos\theta$ is a.

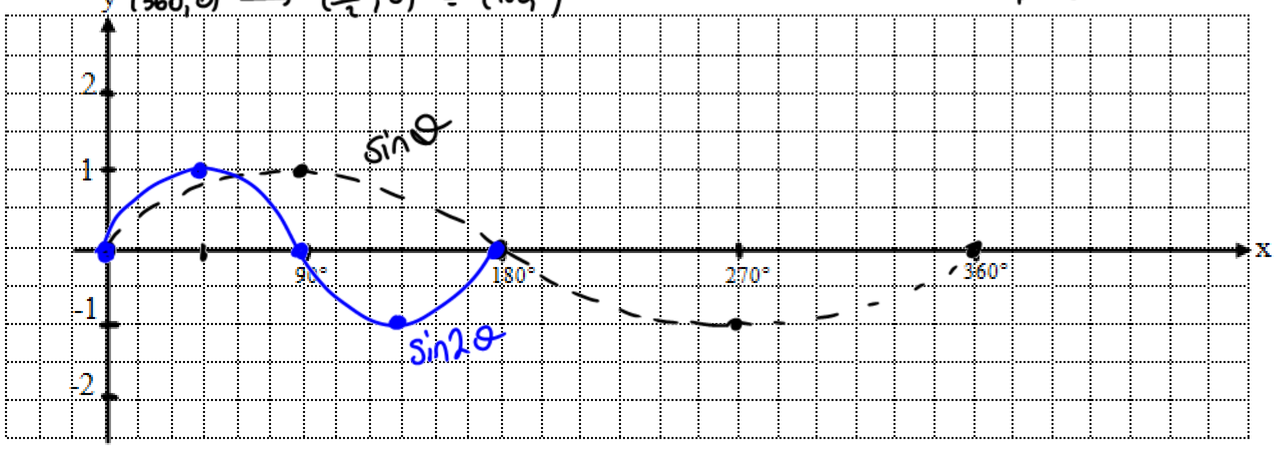
Horizontal Stretches: Investigating for *k*

Recall: $y = f(kx)$ is the image of $y = f(x)$ under a transformation which a causes a **horizontal stretch.**
Mapping: $(x, y) \rightarrow \left(\frac{x}{k}, y\right)$

Example 1: Graph one cycle of $y = \sin\theta$ and $y = \sin 2\theta$ on the grid below using mapping notation.

$(0,0) \longrightarrow (0,0)$
$(90,1) \longrightarrow \left(\frac{90}{2},0\right) = (45,0)$
$(180,0) \longrightarrow \left(\frac{180}{2},0\right) = (90,0)$
$(270,-1) \longrightarrow \left(\frac{270}{2},0\right) = (135,0)$
$(360,0) \longrightarrow \left(\frac{360}{2},0\right) = (180,0)$





For $y = \sin 2\theta$,

1. What coordinate is affected? x

2. What points are unaffected (invariant)? y int.

3. What is the amplitude, a, of the function? 1

4. What is the period? 180

5. What is the equation of the axis of the curve?

$y = 0$

SUMMARY,
Recall: x says something yet does the exact opposite.
for $k > 1$, the graph is horizontally compressed by a factor of $1/k$
for $0 < k < 1$, the graph is horizontally stretched (expanded) by a factor of $1/k$

The value of k determines the number of degrees in the period of the graph. To determine the period of the trigonometric function, divide the period of the base curve by k.

$y = \sin 2\theta$ has period $\frac{360}{k}$ $y = \cos 2\theta$ has period $\frac{360}{k}$

e.g. $y = \sin 2\theta$ has period $\frac{360}{2} = 180$

Ex2: $y = \sin 3\theta$ has period:

$$\text{Period} = \frac{360}{3} = 120^\circ$$

Ex3: $y = \sin\frac{1}{3}\theta$ has period:

$$\text{Period} = \frac{360}{\frac{1}{3}} = 360 \times 3 = \underline{\underline{1080^\circ}}$$

Ex4: Determine the equation of the sine function with amplitude 4 and period 45°. State the domain and range of one cycle.

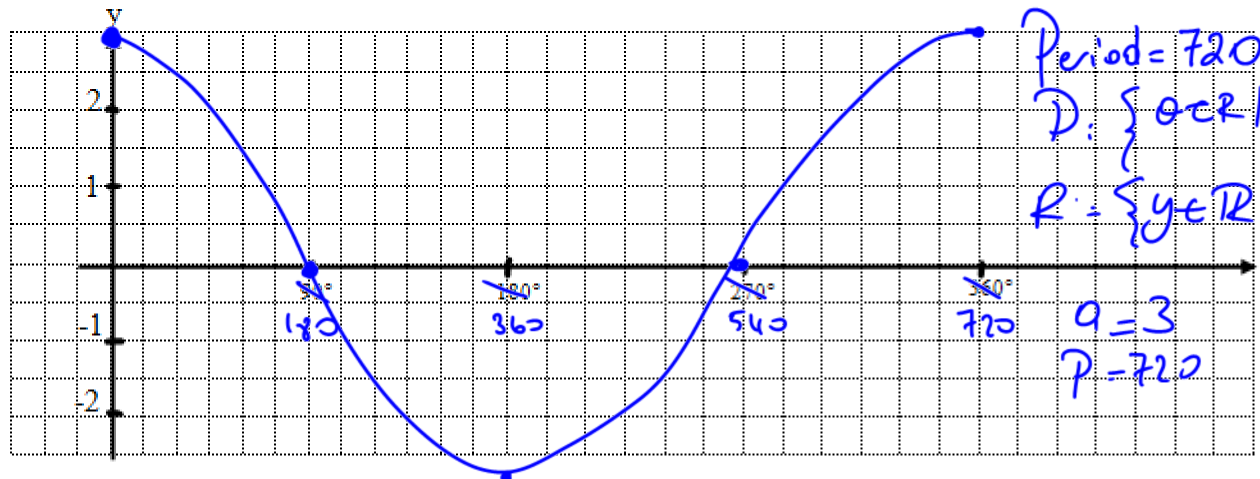
$y = a\sin k\theta$

$\boxed{y = 4\sin 8\theta}$

$\text{Period} = \frac{360}{k}$ $45 = \frac{360}{k}$ $\Rightarrow k = \frac{360}{45} = 8$

$D = \{\theta \in \mathbb{R}\}$ $R = \{y \in \mathbb{R} \mid -4 \leq y \leq 4\}$

Ex5: Sketch one cycle of $y = 3\cos\frac{1}{2}\theta$. State the amplitude, period, domain, and range of one cycle of the function.



Period = 720

$D: \{\theta \in \mathbb{R} \mid 0 \leq \theta \leq 360\}$

$R: \{y \in \mathbb{R} \mid -3 \leq y \leq 3\}$

$a = 3$

$P = 720$

$\cos\theta \longrightarrow 3\cos\frac{1}{2}\theta$

$(x, y) \longrightarrow (2x, 3y)$

$(0, 1) \longrightarrow (2(0), 3(1)) = (0, 3)$

$(90, 0) \longrightarrow (2(90), 3(0)) = (180, 0)$

$(180, -1) \longrightarrow (2(180), 3(-1)) = (360, -3)$

$(270, 0) \longrightarrow (2(270), 3(0)) = (540, 0)$

$(360, -1) \longrightarrow (2(360), 3(-1)) = (720, -3)$