**Ambiguous** is defined as “having more than one possible meaning”. In trigonometry, ambiguity exists for certain problems using the Sine Law.   
**HINT:** Cut pieces of Cappellinis (thin spaghettis) for hands on experience or use a compass.

**1) THE ACUTE CASE: **

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| **CASE 1:** WhenA = 30˚, b = 8 cm, how many triangle(s) can you draw with the missing side that equals to 4 cm? The length of c (side AB) is unspecified (meaning you can change the location of point B). |
| B  b = 8 cm  30o  A  C  Number of triangles constructed  \_\_\_\_\_\_\_\_\_  Did you notice anything special about this line?  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Write an algebraic expression for side a using b and sin A.  **CONCLUSION:**  If A is acute (between 0 and 90) and \_\_\_\_\_\_\_\_\_\_\_\_, there will be only \_\_\_\_\_\_ triangle which is a \_\_\_\_\_\_\_\_ triangle.  **Therefore, no ambiguity exists.** |
| **CASE 2:** How many triangle(s) can you draw if we set the missing side to 3 cm? The length of c (side AB) is unspecified. |
| B  b = 8 cm  30o  A  C  Number of triangles constructed?  \_\_\_\_\_\_\_\_\_\_  Did you notice anything special about this line?  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  **CONCLUSION:**  If A is acute and “a” is less than \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (a < b sin A), there will be \_\_\_\_ triangles. In other words, you cannot draw a triangle with a side that is shorter than the shortest side. |

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| **CASE 3:** How many triangle(s) can you draw if we set the missing side to 8 cm or 9 cm? The length of c (side AB) is unspecified. |
| B  b = 8 cm  30o  A  C  Number of triangles constructed when **a ≥ b** (a is greater than or equal to b)?  \_\_\_\_\_\_\_\_  If A is acute and **a ≥ b**, there will be only **\_\_\_\_\_\_** triangle that can be constructed. **Therefore, no ambiguity exists.** |
| **CASE 4:** How many triangle(s) can you draw if we set the missing side to 5 cm then 7 cm? The length of c (side AB) is unspecified. |
| B  b = 8 cm  30o  A  C  Number of triangles constructed when a is between the height and b (?  \_\_\_\_\_\_\_\_\_\_\_\_  If A is acute and **b sin A < a < b**, there will be \_\_\_\_\_\_\_\_\_\_ triangles that can be constructed.  **Therefore, an AMBIGUOUS Case exists.** |

**2) THE OBTUSE CASE: **

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| **Case 5:**  How many triangles can you draw when the missing side is 3 cm then 4cm? The length of c (side AB) is unspecified.  C  B  A  b =4 cm  150°  If A is obtuse and the missing side is less than side AC (a ≤ b), there will be\_\_\_\_\_\_\_\_triangles. | **Case 6:**  How many triangles can you draw when the missing side is 5cm then 6 cm? The length of c (side AB) is unspecified.  C  B  A  b =4 cm  150°  If A is obtuse and the missing side is greater than side AC (**a > b),** there will be only \_\_\_\_\_triangle.  **Note that there is no ambiguous case if A ≥ 90o** |

When you are given two sides and an angle not in between those sides, you need to be on the lookout for the ambiguous case.

ACUTE TRIANGLE

Determine and the other side using the sine law.

OBTUSE TRIANGLE

Swing the side left to form the obtuse triangle. You have an isosceles triangle. Solve the triangle.

NO  
TRIANGLE

1 RIGHT  
TRIANGLE

2  
TRIANGLES

Between height and given side

Equals to height

Less than height

If the side opposite the given angle is less than the given side, then there is   
NO TRIANGLE.

If the side opposite the given angle is more than the given side, then there is   
ONE TRIANGLE.

Calculate the height to compare it with the side opposite to the given angle.

YES

1 TRIANGLE

NO

ANGLE

ACUTE

OBTUSE

Check if the side opposite the given angle is equal or greater than the given side.

1. In ∆QRS, Q=105º, r=15, q=20. Determine the number of triangles possible. Solve the triangle(s) if possible.
2. In ∆QRS, Q=75º, r=15, q=14. Determine the number of triangles possible. Solve the triangle(s) if possible.
3. In ∆ABC, A = 60º, c = 10cm and a = 9cm. Determine the number of triangles possible. Solve the triangle(s) if possible.