PERIMETER AND AREA RELATIONSHIPS OF A RECTANGLE INVESTIGATION

Optimization is the process of finding values that make a given quantity the greatest (or least) possible given certain conditions.

FIXED PERIMETER

Problem 1: Sarah needs to find the dimensions that will maximize the rectangular area of an enclosure with a perimeter of 24m.

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1	1	11	24	11
2	2	10	24	20
3	3	9	24	27
4	4	8	24	32
5	5	7	24	35
6	6	6	24	36
7	7	5	24	35
8	8	4	24	32
9	9	3	24	27
10	10	2	24	20
11	11	1	24	11

What are the dimensions of the rectangle with the maximum or optimal area? 6 x 6

The maximum area is 36 The shape of the rectangle is square

How can you predict the maximum area if you know the perimeter?

If the shape is a square, it'll maximize the area.

Predict the dimensions of a rectangle with a maximum area that has a perimeter of 60 m:

$P = 4a$

$\frac{60}{4} = \frac{4a}{4}$

$a = 15\ m$

$A = a^2$

$= 15^2$

$= 225\ m^2$

Problem 2: Jeff needs to find the dimensions that will minimize the perimeter of a rectangular enclosure that has an area of 36 m^2.

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Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1	1	36	$2(1+36)=74$	36
2	2	18	$2(2+18)=40$	36
3	3	12	$2(3+12)=30$	36
4	4	9	$2(4+9)=26$	36
5	5	7.2	$2(5+7.2)=24.4$	36
6	6	6	$2(6+6)=24$	36
7	7	5.14	$2(7+5.14)=24.3$	36
8	8	4.5	$2(8+4.5)=25$	36
9	9	4	$2(9+4)=26$	36

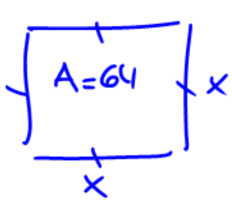
What are the dimensions of the rectangle with the minimum or optimal perimeter? 6 × 6

The minimum perimeter is 24 The shape of the rectangle is square

How can you predict the minimum perimeter if you know the area?

It'll be a square; therefore, I'd find one of the dimensions by square rooting the area. To get the perimeter, I'd multiply it by 4.

Predict the dimensions of a rectangle with a minimum perimeter and an area of 64 m^2:



$A = x^2$

$64 = x^2$

$\boxed{8 = x}$

$P = 4x$

$= 4 \cdot 8$

$= 32\text{ m}.$

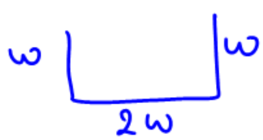
Problem 3: Jessica has 16 m of fencing to enclose a dog pen against the side of a house. She wants to maximize the area for her dog, while using only the 16 m of fencing,

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1	1	14	16	14
2	2	12	16	24
3	3	10	16	30
4	4	8	16	32
5	5	6	16	30
6	6	4	16	24
7	7	2	16	14

What are the dimensions of the rectangle with the maximum or optimal area? 8x 4

The maximum area is 16 The length is 2 times the width.

How can you predict the maximum area if you know the perimeter of an area enclosed on 3 sides?



$P = 4w$

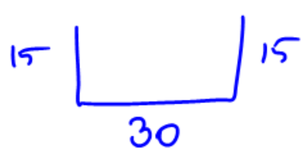
$\frac{P}{4} = w$

Predict the dimensions of a rectangle with maximum area and a perimeter of 60 m, enclosed on only 3 sides. State the dimensions.

$P = 4w$

$60 = 4w$

$\boxed{15 = w}$



$L = 30$

$w = 15$

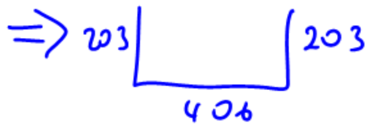
Questions

1. An inbox tray has 3 walls and an open side on one of the longer sides. Determine the maximum area of the tray if all three walls total to a length of 812 mm.

$P = 4w$

$\frac{812}{4} = \frac{4w}{4}$

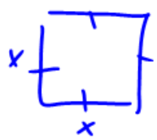
$203 = w$



$A = 203(406)$

$= 82418\,mm$

2. The perimeter of a rectangular piece of cardboard is 46 centimetres. Determine the dimensions that maximize the area.



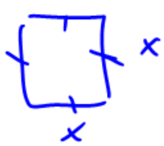
$P = 4x$

$\frac{46}{4} = \frac{4x}{4}$

$\boxed{x = 11.5}$

∴ The dimensions are 11.5×11.5

3. The maximum area of a fenced in pool deck is 1024 m^2. Determine the length of fencing that is required.



$A = x^2$

$1024 = x^2$

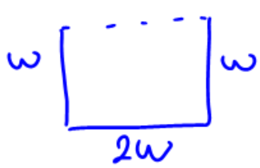
$\boxed{x = 32}$

$P = 4 \cdot 32$

$= 128$

∴ The fencing will be 128m

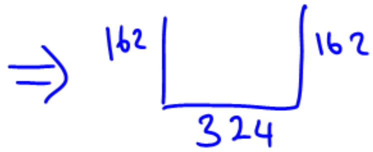
4. Three sides of a look-out deck have a railing, while the fourth side is open. Determine the maximum area if there is 648 cm of railing.



$P = 4w$

$\frac{648}{4} = \frac{4w}{4}$

$w = 162$

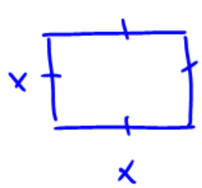


$A = 162(324)$

$= 52488\,cm^2$

∴ The area is $524.88\,m^2$

5. The area of a rectangular box is 722 500 mm^2. Determine the dimensions that minimize the perimeter.



$x^2 = 722500$

$x = 850$

∴ The dimensions are 850×850