

An exponential equation is an equation that contains a variable in the exponent

$$2^x = 4$$

Some exponential equations can be solved **WITHOUT TECHNOLOGY** by writing both sides of the equation as powers of the same base.

KEY WORDS

Exponent
Same base
 $m = n$
exponent
multiple
divide

Equality of Powers with a Common Base

****Called Equating the Powers****

If $a^m = a^n$ then $m = n$ for a is greater than 0 ($a > 0$), a cannot be 1.

For example, since 4^x and 4^3 are both powers of 4, the solution to $4^x = 4^3$ is $x = 3$.

You need to be able to express numbers in exponent form.

Ex: $4 = 2^2$; $9 = 3^2$ or $32 = 2^5$

HOW TO CONVERT INTO EXPONENT FORM

When you need to convert a number into an exponent form. Follow these steps: **ex:** Convert 125 into an exponent

- **Step 1:** Identify if the number is the multiple of 2, 3, 5, etc.
- **Step 2:** Repeatedly divide the number by the number you identified in **Step 1** until you have 1 as a remainder.
- **Step 3:** The divisor is the base; the repetition is the exponent.

Handwritten division for 125:

$$\begin{array}{r|l} 125 & 5 \\ \hline 25 & 5 \\ \hline 5 & 5 \\ \hline 1 & \end{array} \left. \begin{array}{l} \text{divisor} \\ \\ \end{array} \right\} 5^3$$

$125 = 5^3$

TRY: Convert the following numbers into exponent form:

<p>a) 64</p> $\begin{array}{r l} 64 & 2 \\ 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \left. \right\} 2^6$ <p>$64 = 2^6$</p> <p>Check = $2 \wedge 6 = 64$</p>	<p>b) $\frac{1}{81} = 81^{-1} = (3^4)^{-1} = 3^{-4}$ OR $\frac{1}{3^4}$</p> $\begin{array}{r l} 81 & 3 \\ 27 & 3 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array} \left. \right\} 3^4$
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EXAMPLE 1 Finding a Common Base - Solve each of the following. (find the numerical values of x)

<p>a) $5^x = 5^6$</p> <p>$x = 6$</p>	<p>b) $2^x = 32$</p> $\begin{array}{r l} 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \left. \right\} 2^5$ <p>$2^x = 2^5$</p> <p>$x = 5$</p>	<p>c) $2^x = \frac{1}{2}$</p> <p>$2^x = 2^{-1}$</p> <p>$x = -1$</p>	<p>d) $2^x = \frac{1}{4}$</p> <p>$2^x = 4^{-1}$</p> <p>$2^x = (2^2)^{-1}$</p> <p>$2^x = 2^{-2}$</p> <p>$x = -2$</p>	<p>e) $7^{3x-4} = 49$</p> <p>$7^{3x-4} = 7^2$</p> <p>$3x-4 = 2$</p> <p>+4 +4</p> <p>$3x = 6$</p> <p>$x = 2$</p>
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REVIEW

i) Expand $3(x-2)$ using the distributive rule. $3 \cdot x + 3 \cdot (-2) = 3x - 6$

ii) Solve the following equations:

$4x + 8 = 2x$ $\begin{array}{r} -8 \quad -8 \\ 4x = 2x - 8 \\ -2x \quad -2x \\ 2x = -8 \\ \div 2 \quad \div 2 \\ \boxed{x = -4} \end{array}$ <p>You want x on LS #s on RS ① Eliminate +8 by -8 BS ② Eliminate +2x by -2x BS ③ Divide BS by 2</p>	$5x + 3 = 2x - 6$ $\begin{array}{r} -3 \quad -3 \\ 5x = 2x - 9 \\ -2x \quad -2x \\ 3x = -9 \\ \div 3 \quad \div 3 \\ \boxed{x = -3} \end{array}$ <p>① eliminate +3 by -3 BS ② eliminate +2x by -2x BS ③ divide BS by 3</p>	$6(x-2) = 4(2x-1)$ $\begin{array}{r} 6x - 12 = 8x - 4 \\ +12 \quad +12 \\ 6x = 8x + 8 \\ -8x \quad -8x \\ -2x = 8 \\ \div -2 \quad \div -2 \\ \boxed{x = -4} \end{array}$ <p>① use distributive rule BS ② eliminate -12 by +12 on BS ③ eliminate +8x by -8x BS ④ divide BS by -2</p>
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EXAMPLE 2 – Finding a common base – solve each equation

<p>a) $3^{5x+8} = 27^x$</p> $3^{5x+8} = (3^3)^x$ <p>if $\underline{3}^{5x+8} = \underline{3}^{3x}$</p> <p>then $5x + 8 = 3x$</p> $\begin{array}{r} -5x \quad -5x \\ 8 = -2x \\ \div -2 \quad \div -2 \\ -4 = x \\ \therefore x \text{ is } -4 \end{array}$	<p>b) $4^{2(x-4)} = 8^{3x-2}$</p> <p>* express each base in base of 2</p> $(2^2)^{2(x-4)} = (2^3)^{3x-2}$ $2^2 \cdot 2^{2(x-4)} = 2^{3(3x-2)}$ <p>if $\underline{2}^{4(x-4)} = \underline{2}^{3(3x-2)}$</p> <p>then $4(x-4) = 3(3x-2)$ use distributive</p> $\begin{array}{r} 4x - 16 = 9x - 6 \\ -9x \quad -9x \\ -5x - 16 = -6 \\ +16 \quad +16 \\ -5x = 10 \\ \div -5 \quad \div -5 \\ \boxed{x = -2} \end{array}$
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EXAMPLE 3

A cross-country skier forgets a mug of coffee and a muffin in a snow bank. Their temperatures, in degrees Celsius, after t minutes can be modelled by the formulas: $T_{coffee} = 81 \times 3^{-2t}$ and $T_{muffin} = 27 \times 3^{-t}$. Determine when the coffee cools to the same temperature as the muffin.

$$T_{coffee} = T_{muffin}$$

$$81 \times 3^{-2t} = 27 \times 3^{-t} \rightarrow \text{express 81 \& 27 as exponent}$$

$$3^4 \cdot 3^{-2t} = 3^3 \cdot 3^{-t} \rightarrow \text{apply multiplication rule}$$

if $\underline{3}^{4+(-2t)} = \underline{3}^{3+(-t)}$

then $4 - 2t = 3 - t$

$$\begin{array}{r} -3 \quad -3 \\ 1 - 2t = -t \\ +2t \quad +2t \\ \boxed{1 = t} \end{array}$$

\therefore It will take 1 minute.