

An exponential equation is an equation that contains a variable in the exponent

$$2^x = 4$$

Some exponential equations can be solved **WITHOUT TECHNOLOGY** by writing both sides of the equation as powers of the same base.

**KEY WORDS**  
 Exponent  
 Same base  
 $m = n$   
 exponent  
 multiple  
 divide

Equality of Powers with a Common Base

**\*\*Called Equating the Powers\*\***

If  $a^m = a^n$  then  $m = n$  for a is greater than 0 ( $a > 0$ ), a cannot be 1.

For example, since  $4^x$  and  $4^3$  are both powers of 4, the solution to  $4^x = 4^3$  is  $x = 3$ .

You need to be able to express numbers in exponent form.

Ex:  $4 = 2^2$ ;  $9 = 3^2$  or  $32 = 2^5$

**HOW TO CONVERT INTO EXPONENT FORM**

When you need to convert a number into an exponent form. Follow these steps: **ex:** Convert 125 into an exponent

- **Step 1:** Identify if the number is the multiple of 2, 3, 5, etc.
- **Step 2:** Repeatedly divide the number by the number you identified in **Step 1** until you have 1 as a remainder.
- **Step 3:** The divisor is the base; the repetition is the exponent.

Handwritten division for 125:

$$\begin{array}{r|l} 125 & 5 \\ \hline 25 & 5 \\ \hline 5 & 5 \\ \hline 1 & \end{array} \left. \begin{array}{l} \rightarrow \text{divisor} \\ \\ \end{array} \right\} 5^3$$

$125 = 5^3$

**TRY:** Convert the following numbers into exponent form:

<p>a) 64</p> $\begin{array}{r l} 64 & 2 \\ 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \left. \right\} 2^6$ <p><math>64 = 2^6</math></p> <p>Check = <math>2 \wedge 6 = 64</math></p>	<p>b) <math>\frac{1}{81} = 81^{-1} = (3^4)^{-1} = 3^{-4}</math> OR <math>\frac{1}{3^4}</math></p> $\begin{array}{r l} 81 & 3 \\ 27 & 3 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array} \left. \right\} 3^4$
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**EXAMPLE 1 Finding a Common Base** - Solve each of the following. (find the numerical values of x)

<p>a) <math>5^x = 5^6</math></p> <p><u><math>x = 6</math></u></p>	<p>b) <math>2^x = 32</math></p> $\begin{array}{r l} 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \left. \right\} 2^5$ <p><math>2^x = 2^5</math></p> <p><u><math>x = 5</math></u></p>	<p>c) <math>2^x = \frac{1}{2}</math></p> <p><math>2^x = 2^{-1}</math></p> <p><u><math>x = -1</math></u></p>	<p>d) <math>2^x = \frac{1}{4}</math></p> <p><math>2^x = 4^{-1}</math></p> <p><math>2^x = (2^2)^{-1}</math></p> <p><math>2^x = 2^{-2}</math></p> <p><u><math>x = -2</math></u></p>	<p>e) <math>7^{3x-4} = 49</math></p> <p><math>7^{3x-4} = 7^2</math></p> <p><math>3x-4 = 2</math></p> <p>+4 +4</p> <p><math>3x = 6</math></p> <p><u><math>x = 2</math></u></p>
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REVIEW

i) Expand  $3(x-2)$  using the distributive rule.  $3 \cdot x + 3 \cdot (-2) = 3x - 6$

ii) Solve the following equations:

$4x + 8 = 2x$ $\begin{array}{r} -8 \quad -8 \\ 4x = 2x - 8 \\ -2x \quad -2x \\ 2x = -8 \\ \div 2 \quad \div 2 \\ \boxed{x = -4} \end{array}$ <p>You want x on LS #s on RS ① Eliminate +8 by -8 BS ② Eliminate +2x by -2x BS ③ Divide BS by 2</p>	$5x + 3 = 2x - 6$ $\begin{array}{r} -3 \quad -3 \\ 5x = 2x - 9 \\ -2x \quad -2x \\ 3x = -9 \\ \div 3 \quad \div 3 \\ \boxed{x = -3} \end{array}$ <p>① eliminate +3 by -3 BS ② eliminate +2x by -2x BS ③ divide BS by 3</p>	$6(x-2) = 4(2x-1)$ $\begin{array}{r} 6x - 12 = 8x - 4 \\ +12 \quad +12 \\ 6x = 8x + 8 \\ -8x \quad -8x \\ -2x = 8 \\ \div -2 \quad \div -2 \\ \boxed{x = -4} \end{array}$ <p>① use distributive rule BS ② eliminate -12 by +12 on BS ③ eliminate +8x by -8x BS ④ divide BS by -2</p>
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EXAMPLE 2 – Finding a common base – solve each equation

<p>a) <math>3^{5x+8} = 27^x</math></p> $3^{5x+8} = (3^3)^x$ <p>if <math>\underline{3^{5x+8}} = \underline{3^{3x}}</math></p> <p>then <math>5x + 8 = 3x</math></p> $\begin{array}{r} -5x \quad -5x \\ 8 = -2x \\ \div -2 \quad \div -2 \\ -4 = x \\ \therefore x \text{ is } -4 \end{array}$	<p>b) <math>4^{2(x-4)} = 8^{3x-2}</math></p> <p>* express each base in base of 2</p> $(2^2)^{2(x-4)} = (2^3)^{3x-2}$ $2^{2 \cdot 2(x-4)} = 2^{3(3x-2)}$ <p>if <math>\underline{2^{4(x-4)}} = \underline{2^{3(3x-2)}}</math></p> <p>then <math>4(x-4) = 3(3x-2)</math> use distributive</p> $\begin{array}{r} 4x - 16 = 9x - 6 \\ -9x \quad -9x \\ -5x - 16 = -6 \\ +16 \quad +16 \\ -5x = 10 \\ \div -5 \quad \div -5 \\ \boxed{x = -2} \end{array}$
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EXAMPLE 3

A cross-country skier forgets a mug of coffee and a muffin in a snow bank. Their temperatures, in degrees Celsius, after t minutes can be modelled by the formulas:  $T_{coffee} = 81 \times 3^{-2t}$  and  $T_{muffin} = 27 \times 3^{-t}$ . Determine when the coffee cools to the same temperature as the muffin.

$$T_{coffee} = T_{muffin}$$

$$81 \times 3^{-2t} = 27 \times 3^{-t} \rightarrow \text{express 81 \& 27 as exponent}$$

$$3^4 \cdot 3^{-2t} = 3^3 \cdot 3^{-t} \rightarrow \text{apply multiplication rule}$$

if  $\underline{3^{4+(-2t)}} = \underline{3^{3+(-t)}}$

then  $4 - 2t = 3 - t$

$$\begin{array}{r} -3 \quad -3 \\ 1 - 2t = -t \\ +2t \quad +2t \\ \boxed{1 = t} \end{array}$$

$\therefore$  It will take 1 minute.