

An **annuity** is a series of equal payments made at regular intervals. In an **ordinary simple annuity**, payments are made at the end of each compounding period. The **amount of an annuity** is the sum of the regular deposits plus interest.

**EXAMPLE 1 – Using a Table**

Suppose \$450 is deposited at the end of each quarter for 1 year in an investment account that earns 10% per year compounded quarterly.

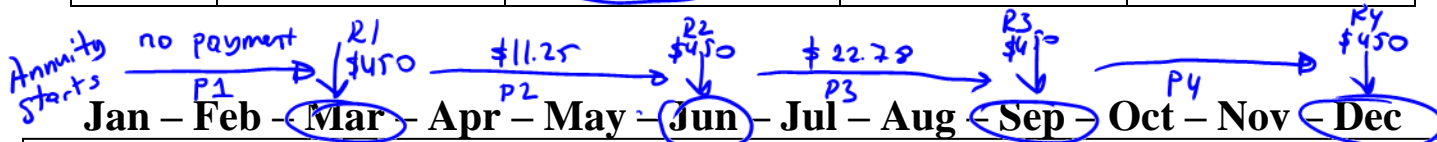
- a) What is the amount of the annuity at the end of 1 year?
- b) How much interest does the annuity earn at the end of 1 year?

**Solution:**

$i = 10\%/1 \text{ year} \div 4 = 0.025$

$n = 1 \text{ year} \times 4 = 4 \text{ periods}$

Quarter	Starting Balance	Interest Earned (0.025)	Deposit	Ending Balance
1	\$0.00	\$0.00	\$450.00	\$450.00
2	\$450	$450 \times 0.025 = \$11.25$	\$450.00	\$911.25
3	\$911.25	\$22.78	\$450.00	\$1384.03
4	\$1384.03	\$34.60	\$450.00	\$1868.63
	Total	<u>\$68.63</u>	$450 \times 4 = \$1800$	



The **AMOUNT** of an ordinary simple annuity is given by the formula  $A = \frac{R[(1+i)^n - 1]}{i}$ , where

$A =$  Final Amount  $i =$  Interest rate in decimal per compounding period

$R =$  Payment per Compounding Period  $n =$  number of compounding periods

This formula can only be used when the **payment interval is the same as the compounding period**

**EXAMPLE 1 – Using a Formula**

- a) What is the amount of the annuity at the end of 1 year?

Type: *compound quarterly*

$A = ?$  *future value*

$R =$  \$450

$i = 10\%/1 \text{ year} = 0.025/4$

$n = 1 \text{ year} \times 4 = 4$

$$A = \frac{450((1+0.025)^4 - 1)}{0.025}$$

$$A = 1868.63$$

$\therefore$  It will grow to \$1868.63.

The **INTEREST** of an ordinary simple annuity is given by the formula  $I = A - Rn$ , where  $I$  is interest amount

- b) How much interest did the annuity earn at the end of 1 year?

Amount = \$1868.63  
Total deposits =  $450 \times 4 = \$1800$

$I = 1868.63 - 1800 = \$68.63$   $\therefore$  Interest earned is \$68.63.

PRACTICE

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

1. Determine the future value of each annuity:

a) \$800 is invested at the end of each month for three years into an account that pays 2.5% per year, compounded monthly.

Type: compound monthly

(FV) A: ?

R: \$800

i: 2.5%/year = 0.025 ÷ 12 = 0.002083

n: 3 years  $\xrightarrow{\times 12}$  36 periods

$$A = \frac{R[(1+i)^n - 1]}{i} = \frac{800((1+0.002083)^{36} - 1)}{0.002083}$$

A = 29837.05

∴ The future value will be \$29837.05

b) \$450 is deposited quarterly for five years into a fund that pays 4.75% per year, compounded quarterly.

Type: compound quarterly

A: ?

R: \$450

i: 4.75%/year  $\xrightarrow{\div 4}$  0.0475 ÷ 4 = 0.011875

n: 5 years  $\xrightarrow{\times 4}$  20 periods

$$A = \frac{R[(1+i)^n - 1]}{i} = \frac{450((1+0.011875)^{20} - 1)}{0.011875}$$

= \$10,091.45

∴ The future value will be \$10,091.45

2. Determine the future value and the interest earned of each annuity.

c) \$800 is deposited biweekly for six years into a fund that pays 3.25% per year, compounded biweekly.

Type: compound biweekly

A: ?

R: \$800

i: 3.25%/year  $\xrightarrow{\div 26}$  0.0325

n: 6 years  $\xrightarrow{\times 26}$  156 periods

$$A = \frac{800((1+0.0025)^{156} - 1)}{0.0025}$$

= 137,704.32

∴ The future value will be \$137,704.32

Amount invested (800 × 156) \$124,800

Interest earned is 137,704.32 - 124,800 = \$12,904.32

d) \$1400 is deposited semi-annually for two years into a fund that pays 7.5% per year, compounded semi-annually.

Type: compound semi-annual

A: ?

R: \$1400

i: 7.5%/year  $\xrightarrow{\div 2}$  0.0375

n: 2 years  $\xrightarrow{\times 2}$  4 periods

$$A = \frac{1400((1+0.0375)^4 - 1)}{0.0375}$$

= 5922.95

∴ The FV will be \$5922.95

Amount invested 1400 × 4 = \$5600

Interest earned is \$322.95