MAP4C **Date: ____________**

Day 6: Future Value of an Annuity **Unit 7 & 8: Financial Math**

An **annuity** is a series of equal payments made at regular intervals. In an **ordinary simple annuity**, payments are made at the end of each compounding period. The **amount of an annuity** is the sum of the regular deposits plus interest.

EXAMPLE 1 – Using a Table

Suppose $450 is deposited at the end of each quarter for 1 year in an investment account that earns 10% per year compounded quarterly.

a) What is the amount of the annuity at the end of 1 year?

b) How much interest does the annuity earn at the end of 1 year?

Solution:

i = 10%/1 year ÷4 = 0.025

n = 1 year x 4 = 4 periods

Quarter	Starting Balance	Interest Earned (0.025)	Deposit	Ending Balance
1	$0.00	$0.00	$450.00	$450.00
2	$450	450 x0.025 = $11.25	$450.00	$911.25
3	$911.25	$22.78	$450.00	$1384.03
4	$1384.03	$34.60	$450.00	$1868.63
	Total	$68.63	450x4 = $1800	

Annuity starts — no payment (P1) → R1 $450 — $11.25 (P2) → R2 $450 — $22.78 (P3) → R3 $450 — (P4) → R4 $450

Jan – Feb – Mar – Apr – May – Jun – Jul – Aug – Sep – Oct – Nov – Dec

The ***AMOUNT*** of an ordinary simple annuity is given by the formula $A = \frac{R\left[(1+i)^n - 1\right]}{i}$, where

A = *Final Amount*

i = *Interest rate in decimal per compounding period*

R = *Payment per Compounding Period*

n = *number of compounding periods*

This formula can only be used when the ***payment interval is the same as the compounding period***

EXAMPLE 1 – Using a Formula

a) What is the amount of the annuity at the end of 1 year?

Type: compound quarterly

A = ? future value

R = $450

i = 10%/1 year = 0.025/q

n = 1year x 4 = 4

$$A = \frac{450\left((1+0.025)^4 - 1\right)}{0.025}$$

$A = 1868.63$

∴ It will grow to $1868.63.

The ***INTEREST*** of an ordinary simple annuity is given by the formula $I = A - Rn$, where I is interest amount

b) How much interest did the annuity earn at the end of 1 year?

Amount = $1868.63

Total deposits = 450 x 4 = $1800

I = 1868.63 − 1800

= $68.63

∴ Interest earned is $68.63.

PRACTICE

1. Determine the future value of each annuity:

$$FV\ A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

a) $800 is invested at the end of each month for three years into an account that pays 2.5% per year, compounded monthly.

Type : compound monthly
(FV) A : ?
R : $800
i : 2.5%/1year = 0.025 ÷ 12 = 0.002083
n : 3 years $\xrightarrow{\times 12}$ 36 periods

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$
$$= \frac{800\left((1+0.002083)^{36} - 1\right)}{0.002083}$$

A = 29837.05

∴ The future value will be $29837.05

b) $450 is deposited quarterly for five years into a fund that pays 4.75% per year, compounded quarterly.

Type : compound quarterly
A : ?
R : $450
i : 4.75%/1year $\xrightarrow{\div 4}$ 0.0475 ÷ 4 = 0.011875
n : 5 years $\xrightarrow{\times 4}$ 20 periods

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$
$$= \frac{450\left((1+0.011875)^{20} - 1\right)}{0.011875}$$

= $10,091.45

∴ The future value will be $10,091.45

2. Determine the future value and the interest earned of each annuity.

c) $800 is deposited biweekly for six years into a fund that pays 3.25% per year, compounded biweekly.

Type : compound biweekly
A : ?
R : $800
i : 3.25%/1 year $\xrightarrow{\div 26}$ 0.00125
n : 6 years $\xrightarrow{\times 26}$ 156 periods

$$A = \frac{800\left((1+0.00125)^{156} - 1\right)}{0.00125}$$

= 137,704.32

∴ The future value will be $137,704.32
Amount invested (800 × 156) $124,800
Interest earn is 137704.32 − 124800 = $12,904.32

d) $1400 is deposited semi-annually for two years into a fund that pays 7.5% per year, compounded semi-annually.

Type : compound semi-annual
A : ?
R : $1400
i : 7.5%/1year $\xrightarrow{\div 2}$ 0.0375
n : 2 years $\xrightarrow{\times 2}$ 4 periods

$$A = \frac{1400\left((1+0.0375)^{4} - 1\right)}{0.0375}$$

= 5922.95

∴ The FV will be $5922.95.
Amount invested 1400 × 4 = $5600
Interest earned is $322.95

Practice: Page 415 #3b, 6bc, 7bc, 9