

Revenue is the product of the price and the number sold

$$\text{REVENUE} = (\text{PRICE})(\text{NUMBER SOLD})$$

Profit is the product of the profit per item and the number sold

$$\text{PROFIT} = (\text{PROFIT PER ITEM})(\text{NUMBER SOLD})$$

### CASE 3: MAXIMIZING REVENUE/ PROFIT

**Ex1.** Rikardo runs a snowboard rental business that charges \$12 per snowboard and averages 36 rentals per day. He discovers that for each \$0.50 decrease in price, his business rents out 2 additional snowboards per day. At what price can Rikardo maximize his revenue?

Revenue = (Price)(number sold)    det "x" be the amount of times Rikardo decreases the price.

$$R = (12 - 0.50x)(36 + 2x)$$

$$= (12)(36) + (12)(2x) - (0.50x)(36x) - (0.50x)(2x)$$

$$= 432 + 24x - 18x - x^2 \quad \text{collect like terms}$$

$$R = -x^2 + 6x + 432 \quad \text{complete the square}$$

$$= -(x^2 - 6x) + 432 \quad \frac{-6}{2} = -3 \quad (-3)^2 = 9$$

$$= -(x^2 - 6x + 9 - 9) + 432$$

$$= -(x^2 - 6x + 9) + 9 + 432 \implies R = -(x-3)^2 + 441$$

$$V(3, 441)$$

x      revenue

$\therefore$  When Ricardo decreases the price 3 times which is  $12 - 0.50(3) = 12 - 1.5 = \$10.5$ , he'll make max rev.

**Ex2.** Calculators are sold to students for 20 dollars each. Three hundred students are willing to buy them at that price. For every 5 dollar increase in price, there are 30 fewer students willing to buy the calculator. What selling price will produce the maximum revenue and what will the maximum revenue be?

Revenue = (Price)(number sold)    det "x" be the amount of increase in price.

$$R = (20 + 5x)(300 - 30x)$$

$$= (20)(300) + (20)(-30x) + (5x)(300) + (5x)(-30x)$$

} expanding

$$R = 6000 - 600x + 1500x - 150x^2$$

$$R = -150x^2 + 900x + 6000$$

$$R = -150(x^2 - 6x) + 6000 \quad \frac{-6}{2} = -3 \quad (-3)^2 = 9$$

$$R = -150(x^2 - 6x + 9 - 9) + 6000$$

$$= -150(x^2 - 6x + 9) + 1350 + 6000$$

$$R = -150(x-3)^2 + 7350$$

$$V(3, 7350)$$

$$\therefore \text{Price} = 20 + 5x$$

$$= 20 + 5(3)$$

$$= 20 + 15$$

$$= \$35$$

Max 7350  
 $\therefore$  at \$35, revenue will be max \$7350

**REVENUE & PROFIT PRACTICE**

1. Computer software programs are sold to students for \$20 each. Three hundred students are willing to buy them at this price. For every \$5 increase in price, there are 30 fewer students willing to buy the software.
  - a) Calculate the selling price that will produce the maximum revenue.
  - b) What is the maximum revenue? (\$15, \$7350)
  
2. Rachel and Ken are knitting scarves to sell at the craft show. The wool for each scarf costs \$6. They were planning to sell the scarves for \$10 each, the same as last year when they sold 40 scarves. However, they know that if they raise the price, they will be able to make more profit, even if they end up selling fewer scarves. They have been told that for every 50¢ increase in the price, they can expect to sell four fewer scarves. What selling price will maximize their profit and what will the profit be? (\$10.50, \$162)
  
3. Last year, a banquet hall charged \$30 per person, and 60 people attended the hockey banquet dinner. This year, the hall's manager has said that for every 10 extra people that attend the banquet, they will decrease the price by \$1.50 per person. What size group would maximize the profit for the hall this year?
  
4. An electronics store sells an average of 60 entertainment systems per month at an average of \$800 more than the cost price. For every \$20 increase in the selling price, the store sells one fewer system. What amount over the cost price will maximize profit?
  
5. You run a canoe-rental business on a small river in Wasaga Beach. You currently charge \$12 per canoe and average 36 rentals a day. An industry journal says that, for every fifty-cent increase in rental price, the average business can expect to lose two rentals a day. Use this information to attempt to maximize your income. What should you charge? (\$10.50)
  
6. Last year the yearbook at Aurora High cost \$75 and only 500 were sold. A student survey found that for every \$5 reduction in price, 100 more students will buy yearbooks. What price should be charged to maximize the revenue from yearbook sales? (\$50)
  
7. A tomato grower needs to ship early when prices are high and spoilage is low. She now has 25 tons on hand and can add two tons a week by waiting. The current profit is \$250 per ton but it will reduce by \$15 per ton for each week she delays. When should she ship to receive maximum profit? ( In about 2 weeks)
 
$$(250 - 15x)(25 + 2x)$$
  
8. A transit company charges \$1.25 per ride and currently averages 10,000 riders per day. The company needs to increase revenue but found that for each \$0.10 increase in fare the company would lose 500 riders. What should the company charge to maximize revenues? (\$0.75 raise to a new fare of \$2.00)