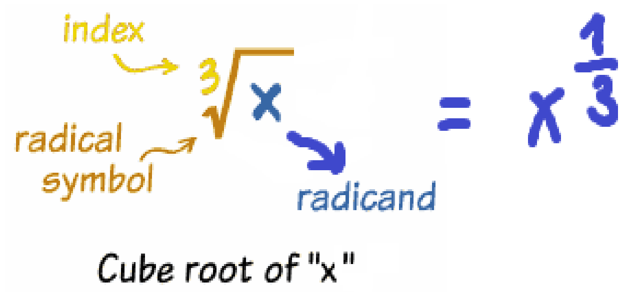
RADICALS



What is a radical?

An expression that has a square root, cube root, etc.

The symbol is $\sqrt{\ }$

Types of Radicals

An **entire radical** is a radical with a coefficient of 1 (e.g.$\sqrt{40}$)

A **mixed radical** has a coefficient other than 1 (e.g.$2\sqrt{5}$).

A. SIMPLIFYING (Reducing) RADICALS

To **simplify** means to find another expression with the same value. It **does not mean** to find a decimal approximation.

METHOD 1: LARGEST PERFECT SQUARE

1. Find the **largest** perfect square which will divide evenly into the number under your radical sign.

Dividend **Divisor**

48	16	the largest perfect square
3	3	that divides evenly into 48 is **16**
1		

2. Write the number appearing under your radical as the product (multiplication) of the perfect square and your answer from dividing.

$$\sqrt{48} = \sqrt{16 \, x \, 3}$$

3. Give each number in the product its own radical sign.

$$\sqrt{48} = \sqrt{16} \, x \, \sqrt{3}$$

4. Reduce the "perfect" radical which you have now created.

$$\sqrt{48} = \sqrt{16 \, x \, 3} = \sqrt{16} \, x \, \sqrt{3} = 4\sqrt{3}$$

5. You now have your answer.

$$\sqrt{48} = 4\sqrt{3}$$

METHOD 2: PRIME FACTORS

1. **Factor** out the number into its prime factors.

48	2
24	2
12	2
6	2
3	3
1	

4 8 = 2 x 2 x 2 x 2 x 3

2. Write all the prime factors under your radical

$$\sqrt{48} = \sqrt{2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 3}$$

3. Give each twin numbers and single numbers in the product their own radical signs

$$\sqrt{48} = \sqrt{2 \text{ x } 2} \text{ x } \sqrt{2 \text{ x } 2} \text{ x } \sqrt{3}$$

4. Reduce the "perfect" radical which you have now created

$$\sqrt{48} = 2 \text{ x } 2\sqrt{3}$$

5. You now have your answer

$$\sqrt{48} = 4\sqrt{3}$$

i) Simplify the following "**entire**" radicals.

a) $\sqrt{40}$ $= \sqrt{4\times10} = \sqrt{4}\times\sqrt{10}$
$= 2\sqrt{5}$

b) $\sqrt{72}$ $= \sqrt{36\times2} = \sqrt{36}\times\sqrt{2} = 6\sqrt{2}$

$180|2$
$90|2$
$45|3$
$15|3$
$5|5$
1

c) $\sqrt{180}$ $= \sqrt{2\times2\times3\times3\times5} = \sqrt{2\times2}\times\sqrt{3\times3}\times\sqrt{5}$
$= 2\times3\times\sqrt{5} = 6\sqrt{5}$

d) $\sqrt{288}$ $= \sqrt{144\times2} = \sqrt{144}\times\sqrt{2} = 12\sqrt{2}$

ii) Express each of the following as "**entire**" radicals.

a) $7\sqrt{5}$ $= \sqrt{49}\times\sqrt{5} = \sqrt{49\times5} = \sqrt{245}$

b) $-3\sqrt{3}$ $= (-1)(3)\sqrt{3} = -\sqrt{9}\times\sqrt{3} = -\sqrt{27}$

B. MULTIPLYING/ DIVIDING RADICALS

When **multiplying** radicals, you must multiply the numbers **OUTSIDE (O)** the radicals **AND** then multiply the numbers **INSIDE (I)** the radicals.

$$O_1\sqrt{I_1}\ x\ O_2\sqrt{I_2} = O_1\ O_2\sqrt{I_1 I_2}\ \text{ such as }\ 2\sqrt{3}\ x\ 4\sqrt{5} = 2x4\sqrt{3x5} = 8\sqrt{15}$$

When **dividing** radicals, you must divide the numbers **OUTSIDE (O)** the radicals **AND** then divide the numbers **INSIDE (I)** the radicals.

$$\frac{O_1\sqrt{I_1}}{O_2\sqrt{I_2}} = \frac{O_1}{O_2}\sqrt{\frac{I_1}{I_2}}\ \text{ such as }\ \frac{4\sqrt{15}}{2\sqrt{3}} = \frac{4}{2}\cdot\sqrt{\frac{15}{3}} = 2\sqrt{5}$$

Rationalizing The Denominator

If a radical appears in the denominator of a fraction, it will need to be "removed" if you are trying to simplify the expression. To "remove" a radical from the denominator, multiply the top and bottom of the fraction by that same radical to create a rational number (a perfect square radical) in the denominator. This process is called ***rationalizing the denominator.***

Simplify $\frac{2}{\sqrt{3}}$

Answer $\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{9}} = \frac{2\sqrt{3}}{3}$

Multiply or divide, then **simplify** the following radicals

a) $2\sqrt{18} \times 3\sqrt{8}$

$= 2\sqrt{9 \times 2} \times 3\sqrt{4 \times 2}$

$= 6\sqrt{2} + 6\sqrt{2}$

$= 36\sqrt{2 \times 2} = 72$

b) $5\sqrt{3} \times 7\sqrt{2}$

$= 5 \times 7\sqrt{3 \times 2}$

$= 35\sqrt{6}$

c) $(\sqrt{3}+5)^2$

$= (\sqrt{3}+5)(\sqrt{3}+5)$

$= 3+5\sqrt{3}+5\sqrt{3}+25$

$= 3+10\sqrt{3}+25$

$= 28+10\sqrt{3}$

d) $\frac{-12\sqrt{24}}{3\sqrt{2}}$

$= \frac{-12}{3}\sqrt{\frac{24}{2}}$

$= -4\sqrt{12}$

$= -4\sqrt{4 \times 3}$

$= -8\sqrt{3}$

e) $\frac{15}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{15\sqrt{5}}{5}$

$= 3\sqrt{5}$

D. ADDING RADICALS

When adding or subtracting radicals, you must use the same concept as that of adding or subtracting "like" variables. **In other words, the radicals must be the same before you add (or subtract) them.**

Like Radicals

example $3\sqrt{6}$ and $2\sqrt{6}$ **non-example** $3\sqrt{6}$ *and* $2\sqrt{5}$

Ex1: Add $3\sqrt{6} + 2\sqrt{6}$	Since the radicals are the same, simply add the numbers **in front** of the radicals (do **NOT** add the numbers under the radicals). **Answer**: $5\sqrt{6}$
Ex2: Add $3\sqrt{6} + 2\sqrt{5}$	Since the radicals are not the same, and both are in their simplest form, there is no way to combine these values. The answer is the same as the problem. **Answer**: $3\sqrt{6} + 2\sqrt{5}$

Add the following radicals

a) $5\sqrt{3} + 2\sqrt{75}$

$= 5\sqrt{3} + 2\sqrt{25 \times 3}$

$= 5\sqrt{3} + 2\sqrt{25}\sqrt{3}$

$= 5\sqrt{3} + 10\sqrt{3}$

$= 15\sqrt{3}$

b) $5\sqrt{8} - 3\sqrt{18} + \sqrt{3}$

$= 5\sqrt{4 \times 2} - 3\sqrt{9 \times 2} + \sqrt{3}$

$= 5\sqrt{4}\sqrt{2} - 3\sqrt{9}\sqrt{2} + \sqrt{3}$

$= 10\sqrt{2} - 9\sqrt{2} + \sqrt{3}$

$= \sqrt{2} + \sqrt{3}$

MULTIPLYING BINOMIALS

1. Simplify. Express your answer as a radical in simplest form.

a) $\sqrt{24}\times\sqrt{18}$

$=\sqrt{6\times4}\times\sqrt{9\times2}$
$=2\sqrt{6}\times3\sqrt{2}$
$=6\sqrt{12}$ $=6\sqrt{4\times3}$
$=12\sqrt{3}$

b) $\sqrt{7}\times\sqrt{8}$

$=\sqrt{7}\times\sqrt{4\times2}$
$=\sqrt{7}\times2\sqrt{2}$
$=2\sqrt{14}$

c) $\sqrt{11}\times\sqrt{14}$

$\sqrt{145}$

d) $\sqrt{8}\times\left(-\sqrt{18}\right)$

$=\sqrt{4\times2}\times(-\sqrt{9\times2}$
$=2\sqrt{2}(-3\sqrt{2})$
$=-6(2)=-12$

e) $\sqrt{20}\times\sqrt{18}$

$=\sqrt{4\times5}\times\sqrt{9\times2}$
$=2\sqrt{5}\times3\sqrt{2}$
$=6\sqrt{10}$

f) $-\sqrt{32}\times\sqrt{72}$

$=-\sqrt{16\times2}\times\sqrt{36\times2}$
$=-4\sqrt{2}\times6\sqrt{2}$
$=-24\sqrt{2\times2}$
$=-48$

g) $\sqrt{20}\times\sqrt{32}\times\sqrt{18}$

$=\sqrt{4\times5}\times\sqrt{16\times2}\times\sqrt{9\times2}$
$=2\sqrt{5}\times4\sqrt{2}\times3\sqrt{2}$
$=24\times2\sqrt{5}$
$=48\sqrt{5}$

h) $\sqrt{24}\times\sqrt{54}\times\sqrt{18}$

$=\sqrt{4\times6}\times\sqrt{9\times6}\times\sqrt{9\times2}$
$=2\sqrt{6}\times3\sqrt{6}\times3\sqrt{2}$
$=6\times6\times3\sqrt{2}$
$=108\sqrt{2}$

2. Simplify. Express your answer as a radical in simplest form.

a) $(3-\sqrt{2})(3+\sqrt{2})$

DOS

$=9+3\sqrt{2}-3\sqrt{2}-\sqrt{2\times2}$
$=9-2$
$=7$

b) $(-7+\sqrt{7})(-7-\sqrt{7})$

DOS

$=(-7)^2-(\sqrt{7})^2$
$=49-7$
$=42$

c) $(\sqrt{11}-x)(\sqrt{11}+x)$

$=(\sqrt{11})^2-(x)^2$
$=11-x$

d) $(\sqrt{5}-3)(3+\sqrt{5})$

$=(\sqrt{5})^2-(3)^2$
$=5-9$
$=-4$

e) $(2+\sqrt{6})(3-\sqrt{10})$

$=6-2\sqrt{10}+3\sqrt{6}-\sqrt{6\times10}$
$=6-2\sqrt{10}+3\sqrt{6}-\sqrt{2\times3\times2\times5}$
$=6+3\sqrt{6}-2\sqrt{10}-2\sqrt{5}$

f) $(-1-\sqrt{15})(8+\sqrt{15})$

$=-8-\sqrt{15}-8\sqrt{15}-15$
$=23-9\sqrt{15}$

g) $(x+1+\sqrt{2})(x+1-\sqrt{2})$

$=(x+1)^2-(\sqrt{2})^2$
$=x^2+2x+1-2$
$=x^2+2x-1$

h) $(y-\sqrt{55})(y+\sqrt{22})$

$=y^2+y\sqrt{22}-y\sqrt{55}-\sqrt{5\times11\times2\times11}$
$=y^2+y\sqrt{22}-y\sqrt{55}-11\sqrt{10}$

1a) $12\sqrt{3}$	b) $2\sqrt{14}$	c) $\sqrt{154}$	d) -12
e) $6\sqrt{10}$	f) -48	g) $48\sqrt{5}$	h) $108\sqrt{2}$
2a) 7	b) 42	c) $11-x^2$	d) -4
e) $6+3\sqrt{6}-2\sqrt{10}-2\sqrt{15}$	f) $-23-9\sqrt{15}$	g) x^2+2x-1	h) $y^2+(\sqrt{22}-\sqrt{55})y-11\sqrt{10}$

SIMPLIFYING RADICALS

1. Simplify:

$450 \mid 9$
$50 \mid 25$
2

a) $\sqrt{12} = \sqrt{4\times3} = 2\sqrt{3}$ b) $\sqrt{20} = \sqrt{4\times5} = 2\sqrt{5}$ c) $\sqrt{45} = \sqrt{9\times5} = 3\sqrt{5}$
d) $\sqrt{450} = \sqrt{9\times25\times2} = 15\sqrt{2}$ e) $\sqrt{72} = \sqrt{36\times2} = 6\sqrt{2}$ f) $\sqrt{200} = \sqrt{100\times2} = 10\sqrt{2}$

2. Simplify:

a) $\frac{\cancel{12}^{4}\sqrt{2}}{\cancel{3}} = 4\sqrt{2}$ b) $\frac{\cancel{6}\sqrt{6}}{\cancel{6}} = \sqrt{6}$ c) $\frac{\cancel{10}^{5}\sqrt{11}}{\cancel{12}_{6}} = \frac{5\sqrt{11}}{6}$ d) $\sqrt{\frac{100}{144}} = \sqrt{\frac{25}{36}} = \frac{5}{6}$

3. Solve for x.

a) $\sqrt{x^2} = \sqrt{169}$
$x = \mp 13$

b) $\frac{\cancel{3}x^2}{} = \cancel{144}$, $\sqrt{48}$
$x = \mp 4\sqrt{3}$

c) $\sqrt{(x-4)^2} = \sqrt{81}$
$(x-4) = \mp 9$
$x-4 = 9$, $x = 13$
$x-4 = -9$, $x = -5$

d) $\frac{2(x+1)^2}{2} = \frac{18}{2} \Rightarrow \sqrt{(x+1)^2} = \sqrt{9}$
$(x+1) = \mp 3$
$x = -4$
$x = 2$

4. Express both roots in decimal form, rounded to 3 decimal places.

a) $x = 2 \pm 2\sqrt{2}$ b) $x = -2 \pm \frac{\sqrt{5}}{3}$ c) $x = -\frac{7}{5} \pm \frac{2\sqrt{10}}{5}$ $\frac{3\sqrt{8}+\sqrt{12}}{2}$

5. Simplify:

a) $2\sqrt{2}+\sqrt{27}+2\sqrt{12}+\sqrt{8}$
b) $\frac{\sqrt{24}\sqrt{8}}{\sqrt{3}}$
c) $\frac{\sqrt{6}\sqrt{10}}{\sqrt{12}}$
d) $\frac{6+\sqrt{12}}{2}$
e) $(\sqrt{2}+3)(\sqrt{2}-3)$
f) $(3\sqrt{2}+4\sqrt{3})(3\sqrt{2}-4\sqrt{3})$
g) $2\sqrt{24}(-3\sqrt{3})$
h) $\frac{15\sqrt{6}}{5\sqrt{12}}$
i) $2\sqrt{6}(3\sqrt{2}-\sqrt{3})$
j) $(6+4\sqrt{3})(6-4\sqrt{3})$

Answers

1. a) $2\sqrt{3}$ b) $2\sqrt{5}$ c) $3\sqrt{5}$ d) $15\sqrt{2}$ e) $6\sqrt{2}$ f) $10\sqrt{2}$

2. a) $4\sqrt{2}$ b) $\sqrt{6}$ c) $\frac{5\sqrt{11}}{6}$ d) $\frac{5}{6}$

3. a) ±13 b) $\pm4\sqrt{3}$ c) $-5, 13$ d) $-4, 2$

4. a) $-0.828, 4.828$ b) $-2.745, -1.255$ c) $-2.665, -0.135$

5. a) $4\sqrt{2}+7\sqrt{3}$ b) 8 c) $\sqrt{5}$ d) $3+\sqrt{3}$
e) - 7 f) - 30 g) $-36\sqrt{2}$ h) $\frac{3}{\sqrt{2}}$
i) $12\sqrt{3}-6\sqrt{2}$ j) - 12

5. Simplify:

a) $2\sqrt{2}+\sqrt{27}+2\sqrt{12}+\sqrt{8}$

b) $\dfrac{\sqrt{24}\sqrt{8}}{\sqrt{3}}$

c) $\dfrac{\sqrt{6}\sqrt{10}}{\sqrt{12}}$

d) $\dfrac{6+\sqrt{12}}{2}$

e) $(\sqrt{2}+3)(\sqrt{2}-3)$

f) $(3\sqrt{2}+4\sqrt{3})(3\sqrt{2}-4\sqrt{3})$

g) $2\sqrt{24}(-3\sqrt{3})$

h) $\dfrac{15\sqrt{6}}{5\sqrt{12}}$

i) $2\sqrt{6}(3\sqrt{2}-\sqrt{3})$

j) $(6+4\sqrt{3})(6-4\sqrt{3})$

5a) $2\sqrt{2}+\sqrt{9\times 3}+2\sqrt{4\times 3}+\sqrt{4\times 2}$

$=2\sqrt{2}+\sqrt{9}\sqrt{3}+2\sqrt{4}\sqrt{3}+\sqrt{4}\sqrt{2}$

$=2\sqrt{2}+3\sqrt{3}+(2)(2)\sqrt{3}+2\sqrt{2}$

$=2\sqrt{2}+3\sqrt{3}+4\sqrt{3}+2\sqrt{2}$

$=4\sqrt{2}+7\sqrt{3}$

b) $\dfrac{\sqrt{4\times 6}\sqrt{4\times 2}}{\sqrt{3}}=\dfrac{\sqrt{4}\sqrt{6}\sqrt{4}\sqrt{2}}{\sqrt{3}}=\dfrac{4\sqrt{12}}{\sqrt{3}}$

$=4\sqrt{\dfrac{12}{3}}=4\sqrt{4}=4(2)=8$

e) $(\sqrt{2}+3)(\sqrt{2}-3)$

$=(\sqrt{2})(\sqrt{2})-\cancel{3\sqrt{2}}+\cancel{3\sqrt{2}}-9$

$=2-9=\underline{-7}$

f) $(3\sqrt{2}+4\sqrt{3})(3\sqrt{2}-4\sqrt{3})$

$=9\sqrt{2\times 2}-12\sqrt{2\times 3}+12\sqrt{3\times 2}-16\sqrt{3\times 3}$

$=9(2)-\cancel{12\sqrt{6}}+\cancel{12\sqrt{6}}-16(3)$

$=18-48$

$=-30$

Did you also notice it was difference of squares

$(3\sqrt{2})^2-(4\sqrt{3})^2=9(2)-16(3)$

$=-30$

Math is Radical

196 | 2
98 | 2
49 | 7

150 | 2
75 | 25
3 | 3

1. $\sqrt{196} = \sqrt{2\times2\times7\times7} = 14$	2. $\sqrt{20} = \sqrt{4\times5} = 2\sqrt{5}$	3. $\sqrt{75} = \sqrt{25\times3} = 5\sqrt{3}$	4. $\sqrt{150} = \sqrt{25\times6} = 5\sqrt{6}$	5. $\sqrt{48} = \sqrt{16\times3} = 4\sqrt{3}$
6. $2\sqrt{5}\cdot8\sqrt{3} = 16\sqrt{15}$	7. $4\sqrt{2}\cdot3\sqrt{14} = 12\sqrt{28} = 12\sqrt{4\times7} = 24\sqrt{7}$	8. $(-12\sqrt{2})\cdot(-5\sqrt{6}) = 60\sqrt{12} = 60\sqrt{4\times3} = 120\sqrt{3}$	9. $3\sqrt{5}\cdot7\sqrt{5} = 21\sqrt{25} = 21(5) = 105$	10. $4\sqrt{2}\cdot3\sqrt{10} = 12\sqrt{20} = 12\sqrt{4\times5} = 24\sqrt{5}$
11. $\frac{4\sqrt{14}}{8\sqrt{7}} = \frac{\sqrt{14}}{2\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{14\times7}}{2\cdot7} = \frac{\sqrt{7\times7\times2}}{14} = \frac{7\sqrt{2}}{14} = \frac{\sqrt{2}}{2}$	12. $-\frac{12\sqrt{6}}{3\sqrt{2}} = -4\sqrt{\frac{6}{2}} = -4\sqrt{3}$	13. $\frac{4\sqrt{18}}{3\sqrt{2}} = \frac{4}{3}\sqrt{\frac{18}{2}} = \frac{4}{3}\sqrt{9} = \frac{4}{3}\cdot3 = 4$	14. $-\frac{5\sqrt{60}}{2\sqrt{3}} = -\frac{5}{2}\sqrt{\frac{60}{3}} = -\frac{5}{2}\sqrt{20} = -\frac{5}{2}\sqrt{4\times5} = -\frac{5}{2}(2)\sqrt{5} = -5\sqrt{5}$	15. $\frac{3\sqrt{150}}{5\sqrt{3}} = \frac{3}{5}\sqrt{\frac{150}{3}} = \frac{3}{5}\sqrt{50} = \frac{3}{5}\sqrt{25\times2} = \frac{3}{5}(5)\sqrt{2} = 3\sqrt{2}$
16. $\frac{14\sqrt{12}}{7\sqrt{24}} = 2\sqrt{\frac{12}{24}} = \frac{2\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$	17. $\frac{4\sqrt{5}}{2\sqrt{15}} = 2\sqrt{\frac{5}{15}} = 2\sqrt{\frac{1}{3}} = \frac{2\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{2\sqrt{3}}{3}$	18. $\frac{10\sqrt{10}}{15\sqrt{15}} = \frac{2}{3}\sqrt{\frac{10}{15}} = \frac{2\sqrt{2}}{3\sqrt{3}}$; $\frac{2\sqrt{2}\sqrt{3}}{3\sqrt{3}\sqrt{3}} = \frac{2\sqrt{6}}{9}$	19. $4\sqrt{\frac{1}{2}} = \frac{4\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$	20. $5\sqrt{\frac{6}{18}} = 5\sqrt{\frac{1}{3}} = \frac{5\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{5\sqrt{3}}{3}$
21. $-2\sqrt{3} + 14\sqrt{3} = 12\sqrt{3}$	22. $3\sqrt{8} + 3\sqrt{2} = 3\sqrt{4\times2} + 3\sqrt{2} = 6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$	23. $-8\sqrt{3} + 4\sqrt{27} = -8\sqrt{3} + 4\sqrt{9\times3} = -8\sqrt{3} + 12\sqrt{3} = 4\sqrt{3}$	24. $5\sqrt{50} + 2\sqrt{8} - 8\sqrt{2} = 5\sqrt{25\times2} + 2\sqrt{4\times2} - 8\sqrt{2} = 25\sqrt{2} + 4\sqrt{2} - 8\sqrt{2} = 21\sqrt{2}$	25. $\sqrt{48} - 4\sqrt{27} + 2\sqrt{75} = \sqrt{16\times3} - 4\sqrt{9\cdot3} + 2\sqrt{25\cdot3} = 4\sqrt{3} - 12\sqrt{3} + 10\sqrt{3} = 2\sqrt{3}$

Answers (jumbled):

$2\sqrt{2}$ $12\sqrt{3}$ $120\sqrt{3}$ $3\sqrt{2}$ $5\sqrt{6}$ $24\sqrt{7}$ 105 $\frac{\sqrt{2}}{2}$ 4 $-5\sqrt{5}$ $4\sqrt{3}$ $\sqrt{2}$ $\frac{2\sqrt{3}}{3}$

$2\sqrt{5}$ $\frac{5\sqrt{3}}{3}$ $-4\sqrt{3}$ $5\sqrt{3}$ $21\sqrt{2}$ $16\sqrt{15}$ $4\sqrt{3}$ $2\sqrt{3}$ $9\sqrt{2}$ 14 $24\sqrt{5}$ $\frac{2\sqrt{6}}{9}$