Real data rarely fits a mathematical model perfectly. We need to find a model that best fits the data collected.

To help decide which model might be best to use when modeling data, you can:

1. Examine the first and second differences and the growth/decay factors
2. Create a scatter plot of the data
3. Look at the R^2 value of a regression model. R^2 is a statistical measure of how close the data are to the fitted regression line. R^2 is always between 0 and 100%.

KEY WORDS
Perfectly
best
first
second
differences
growth/decay
scatter
regression
statistical
100

Identifying Relationships in Data

EXAMPLE 1 Electrical appliances such as a VCR or digital clock contain a capacitor for power during brief electrical outages. The table shows how the voltage in a capacitor decreases over time after a power outage.

Which type of relationship seems to exist between voltage and time? Justify your answer.

a) Determine the 1st, 2nd differences and Decay factor since the Voltage levels are decreasing.

not LINEAR NOT QUADRATIC exponential

Time (s)	Voltage (V)	1st Differences	2nd Differences	Decay Factor
0	9.0	7 – 9 = -2		7 / 9 = 0.78
2	7.0	5.2-7=-1.8	-1.8-(-2)=0.2	5.2/7=0.74
4	5.2	3.9-5.2=-1.3	-1.3-(-1.8)=0.5	3.9/5.2=0.75
6	3.9	3-3.9=0.9	-0.9-(-1.3)=0.4	3/3.9=0.77
8	3.0	2.3-3=-0.7	-0.7-(-0.9)=0.2	2.3/3=0.77
10	2.3			

The 1st differences are ~~not~~ the same; therefore, it is NOT **linear relationship**.

The 2nd differences are ~~not~~ the same; therefore, it is NOT **quadratic relationship**.

Decay factor is almost the same; therefore, it is an **exponential relationship**.

EXAMPLE 2: In a science experiment, students punched a hole near the bottom of a 2-L pop bottle. They filled the bottle with water and measured how the water level changed over time. The results are shown in the table below.

x- axis Time (s)	0	25	50	75	100
y-axis Water Level (cm)	30.0	22.3	16.1	11.2	7.8

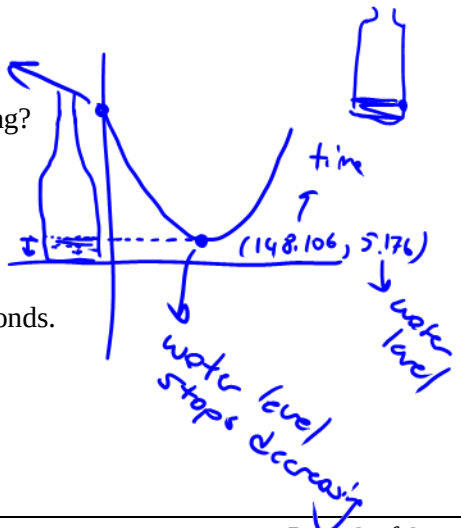
Using **DESMOS graphing calculator (either download or go to desmos.com)**, find a model for the data:

- ☑ Click ➕, then choose table.
- ☑ Using the table above, enter the values for time in the x_1 column and values for water level in the y_1 column.
- ☑ To see the scatter plot, zoom out.
- ☑ Click on the 2nd line below your table to enter the following:
- ☑ Type y1~mx1+b. This is how it should look like on your screen $y_1 \sim mx_1 + b$
- ☑ In the table below record r^2, this is called regression; calculate the percentage by multiplying r^2 by 100.
- ☑ Record m and b values, finally form your equation by substituting the numerical values for m and b.
- ☑ We are done with linear model. Now let's enter the values for quadratic model to see if the data fits the quadratic model more than the linear one.
- ☑ Type y1~ax1² + bx1² + c. It should look like this: $y_1 \sim ax_1^2 + bx_1 + c$
- ☑ In the table below, record r^2; calculate percentage, and write the equation.
- ☑ Finally, we will check the exponential modal.
- ☐ Type y1~ab^x1. It should look like this: $y_1 \sim ab^{x_1}$
- ☐ Write the equation in the table below.

	LINEAR	**QUADRATIC (Polynomial)**	**EXPONENTIAL**
r^2	0.9778	1	0.9981
Percentage r^2 x 100	0.9778 x 100 = 97.78%.	100%.	99.81%.
Equation	m = -0.222 b = 28.58 y = mx + b y = -0.222x + 28.58	a = 0.00113143 b = -0.335143 c = 29.9943 y = ax² + bx + c y = 0.00113143x² - 0.335143x + 29.9943	a = 30.3032 b = 0.987029 y = ab^x y = 30.3032(0.987029)^x

c) Which model best represents the data? Justify your answer.

Quadratic model best represents the data because regression model is 100%.



d) Examine the graph and determine how long it takes for the water to stop flowing?

apx. 148 sec

e) What was the initial height of the water? 30 cm

f) What is the height of the hole? 5.176 cm

g) Using the formula, calculate the height of the water level when time is 120 seconds.

$y = 0.00113143x^2 - 0.335143x + 29.9943$

$y = 0.00113143 \cdot (120)^2 - 0.335143(120) + 29.9943$

$y = 6.069732$ ∴ Water level is 6.07cm.